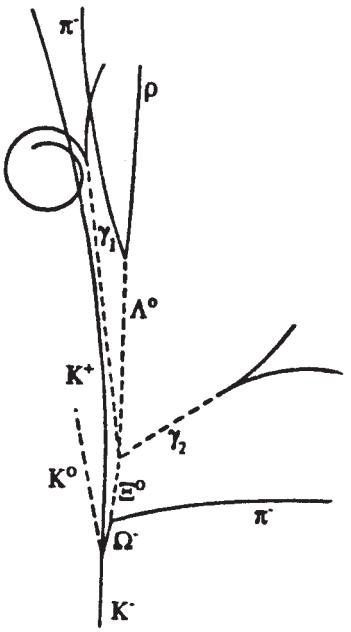
VOLUME 37, NUMBER 1, FEBRUARY 2014

HADRONIC JOURNAL Volume 37, Number 1, February 2014

HADRONIC JOURNAL

Founded in 1978 by Prof. R. M. Santilli at Harvard University. Some of the past Editors include Professors S. L. Adler, A. 0. Barut, L. C. Biedenharn, N. N. Bogoliubov, M. Froissart, J.Lohmus, S. Okubo, Nobel Laureate Ilya Prigogine, M. L. Tomber, J. P.Vigier, J. Wess, Nobel Laureate Chen Ning Yang.



EDITORIAL BOARD
A.O.Animalu
D. P. Bhattacharyya
G.-C. Cavalleri
V. De Sabbata
T. L. Gill
L. P. Horwitz
H. Hurst
S. L. Kalla
S. I. Kruglov
S. S. Moskaliuk
M. Nishioka
R. F. O'Connell
Z. Oziewicz
E. Recami
M. Saleem
D. V. Shirkov
S. Silvestrov
D.S. Sourlas
H. M. Srivastava
A. I. Studenikin
R.I. Tsonchev
QI-Ren Zhang
C.A.Wolf
YI-Zhong Zhuo

FOUNDER:
RUGGERO M. SANTILLI

EDITOR IN CHIEF
CHRISTIAN CORDA

HADRONIC PRESS, INC.

HADRONIC JOURNAL

VOLUME 37, NUMBER 1, FEBRUARY 2014

TRAJECTORIES OF ANTIMATTER BODIES IN OUR SOLAR SYSTEM ACCORDING TO ISODUAL THEORY OF ANTIMATTER AND POSSIBLE IMPACTS WITH EARTH, 1

Simone Beghella Bartoli
The Institute for Basic Research
1444 Rainville Rd., Tarpon Springs, FL 34689 USA

THE MATRIX FIELD THEORY AND THE GENERALIZED SCHWARZSCHILD SOLUTION OF THE METRIC MATRIX EQUATION, 29

Alexander D. Dymnikov
Louisiana Accelerator Center/Physics Department
Louisiana University at Lafayette
320 Cajundome Blvd., Lafayette, LA 70506 USA

DYSON'S ANALYSIS FOR TOKAMAK GRAVITY WAVE PRODUCTION, LIGO, AND GERTSENSHTEIN COUPLING BETWEEN PHOTONS, GRAVITONS AND MAGNETIC FIELDS, 57

Andrew Beckwith
Chongqing University Department of Physics
Chongqing, PRC, 400044

COMMENT ON "WHAT THE INFORMATION LOSS IS NOT", 75

Qing-yu Cai, Baocheng Zhang, Miangsheng Zhan
Wuhan Institute of Physics and Mathematics
Chinese Academy of Sciences, Wuhan 430071
People's Republic of China
Li You
Department of Physics, Tsinghua University
People's Republic of China

THE SUBSTRUCTURE OF THE QUANTUM FIELD-OSCILLATOR, 83

Guido Giovanni
Department of Mathematics and Physics
High School "C. Cavalleri"
Via Spangliardi 23, Parabiago, 20015 Italy

ADDENDUM ERRATA CORRIGE
INDEX VOL 36, 2013, 125

TRAJECTORIES OF ANTIMATTER BODIES IN OUR SOLAR SYSTEM ACCORDING TO ISODUAL THEORY OF ANTIMATTER AND POSSIBLE IMPACTS WITH EARTH[1]

Simone Beghella Bartoli
The Institute for Basic Research
1444 Rainville Rd.
Tarpon Springs, FL 34689, U.S.A.
sibebart@gmail.com

Received February 5, 2014
Revised April 18, 2014

Abstract

R. M. Santilli has proposed the innovative isodual theory of antimatter for the classical representation of neutral or charged antimatter bodies in a way compatible with available classical experimental evidence on antimatter. Santilli's isodual theory is equivalent to charge conjugation at the operator level, thus being compatible with experimental data on antimatter also at the particle level. A first prediction of the isodual theory holding at all levels of study, from the isodual Newtonian mechanics to the isodual Riemannian geometry, is that matter and antimatter experience a gravitational repulsion (antigravity) due to the negative value of the curvature tensor requested by matter-antimatter conjugation. A second prediction of the isodual theory is the existence of antimatter galaxies and, of course, antimatter asteroids traveling in space, that could represent a threat to our planet. Since Earth appears to have been hit in the past by antimatter asteroids (as it seems to be the case for the 1908 Tunguska explosion in Siberia), the author presents in this paper, apparently for the first time, a study of possible trajectories that would lead an antimatter asteroid to collide with our planet under the assumption of the repulsive gravitational interaction between Earth and the antimatter asteroid. Moreover, an estimate of the minimum approach speed required for an antimatter asteroid to impact with Earth is theoretically and numerically identified at different distances from our planet, along with other considerations about the trajectories.

---

[1] Supported in part by the R. M. Santilli Foundation

Copyright © 2014 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. Introduction

Having been conceived before the discovery of antimatter, Newtonian mechanics, Galileian relativity and Einsteinian special and general relativities are not able to give a classical representation of neutral antimatter. As a consequence, matter and antimatter are commonly believed to differ only in the sign of the charge in contradiction with the annihilation process, quantization and other aspects. This occurrence created a scientific imbalance , because throughout the 20th century matter was studied at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely studied at the particle level (see monograph [1] for historical references and technical details).

In order to resolve this scientific imbalance, R. M. Santilli has proposed the innovative isodual theory of antimatter for the classical representation of neutral or charged antimatter bodies in a way compatible with available classical experimental evidence on antimatter. Santilli's isodual theory is equivalent to charge conjugation at the operator level, thus being compatible with experimental data on antimatter also at the particle level (see [1] for more details).

A second prediction of the isodual theory is the existence of antimatter galaxies that appear to be confirmed by recent astrophysical observations based on a new telescope with concave lenses [2] and preliminarily confirmed by independent scholars in ref. [3].

The existence of antimatter galaxies evidently implies the existence of antimatter supernova and, therefore, of antimatter asteroids traveling through the universe, with consequential danger for our planet (see Ref. [2] and papers quoted therein).

Since Earth appears to have been hit in the past by antimatter asteroids (as it seems to be the case for the 1908 Tunguska explosion in Siberia, see [4]), the author presents in this paper, apparently for the first time, a study of possible trajectories that would lead an antimatter asteroid to collide with our planet under the assumption of the repulsive gravitational interaction between Earth and the antimatter asteroid.

Other considerations are also made about the nature of these trajectories, the minimum velocity needed to impact Earth (since the Sun's gravity would repulse an approaching antimatter body slowing it down) and the minimum angle between the direction of arrival and the Sun at which an antimatter asteroid can impact Earth without being deviated. These aspects are investigated both theoretically and numerically, using an optimization process analogous to that used by the author in preceding work [5].

2. Modelization

To model the trajectory of an antimatter asteroid, we have to consider Newton's law of universal gravitation, adapting it to our case. For the generic n-th body of the Solar System, we can write:

$$F_n = G \cdot \frac{M_n \cdot M_{ast}^d}{r^2} = -G \cdot \frac{M_n \cdot M_{ast}}{r^2} \qquad (1)$$

where:

- F_n is the force applied to the antimatter asteroid by the gravity of the n-th body considered
- G is the gravitational constant
- M_n is the mass of the n-th body considered
- M^d_{ast} is the isodual mass of the antimatter asteroid
- r is the distance between the two bodies (considered point-like)

Hence the vectorial form:

$$\boldsymbol{F}_n = -G \frac{M_n M_{ast}^d}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) = G \frac{M_n M_{ast}}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) \qquad (2)$$

$$M_{ast} \left(\frac{d^2 \boldsymbol{r}}{dt^2} \right)_n = G \frac{M_n M_{ast}}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) \qquad (3)$$

where:

- $\boldsymbol{F}_n$ is the 3-components force vector
- $\boldsymbol{r}$ is the 3-components vector representing the distance of the asteroid from a fixed reference frame
- $\boldsymbol{r}_n$ is the 3-components vector representing the distance of the n-th body considered from a fixed reference frame

At this point, we must consider the effect of all the major bodies of the Solar System at the same time, making a summation of the forces due to every celestial body:

$$\frac{d^2 \boldsymbol{r}}{dt^2} = G \cdot \sum_{n=1}^{10} M_n \frac{\boldsymbol{r} - \boldsymbol{r}_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \qquad (4)$$

For the calculations all the planets were considered plus the Sun and Pluto.
In order to determine the state of the asteroid (position and velocity) at every time, we need then to solve a system of 6 non-linear differential equations:

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{Bmatrix} = \begin{Bmatrix} v_x \\ v_y \\ v_z \\ \sum_{n=1}^{10} m_n \frac{x_. - x_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \\ \sum_{n=1}^{10} m_n \frac{y - y_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \\ \sum_{n=1}^{10} m_n \frac{z - z_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \end{Bmatrix} \qquad (5)$$

The positions of the planets in this case are known data taken from available ephemeris, since the mass of the asteroid, and therefore its gravitational effect on the bodies of the Solar System, is considered negligible, so only the effects of planets on the asteroid are computed.

This system of equations, due to its complexity, can be solved only via numerical integration.
By defining a starting condition, in this case the initial sate of the asteroid, and the duration of the propagation, we can integrate the equations and obtain a trajectory in space.

At this point, with the choice of an initial time and relative initial state, we can propagate the trajectory forward in time and obtain a unique solution, but this approach makes it extremely difficult to find collisions with Earth, since *a priori* we have no idea where the initial conditions will lead the asteroid.

Another more useful approach is to give the program the final conditions, in terms of position and time (using of course the position of Earth at that time) and integrate backwards in time, in order to obtain the initial state at another arbitrary time. This way, we still have the 3 components of the final speed as degrees of freedom to obtain different trajectories in space.

It is correct to use this approach within the limits of model used, since in our approximation all forces are conservative, so the motion itself is reversible in time

without violating physical laws. In the real world, we would have other interactions, such as those with solar wind and sunlight, but these interactions have not been included at this level, due to their assumed small entity and to still unsolved problems about the effects of matter light hitting an antimatter body (see paragraph 5).

3. Examples of trajectories

Here we present some examples of trajectories obtained with the above presented equations from arbitrary conditions. For clarity's sake only inner planets are represented (Sun, Mercury, Venus, Earth and Mars), while outer planets are considered in calculations but not represented in these graphs, to preserve readability.

In *Figure 1,* we have a comparison between the forward propagated trajectories of a matter asteroid and an antimatter asteroid, starting at the same position, with the same velocity and at the same time.
We can see that the two trajectories are completely different, since the matter asteroid turns left and passes behind the Sun, while the antimatter one is deflected to the right and escapes the Solar System without getting even close to the Sun.

In Figures *Figure* 2 toFigure 4 we have examples of forward propagated trajectories of antimatter asteroids in our Solar System, both close to and outside the ecliptic plane.

In Figures *Figure* 5 to *Figure* 8 we have examples of backward propagated trajectories of antimatter asteroids, whose ending position is fixed at Earth, while the impact velocity is arbitrarily chosen.

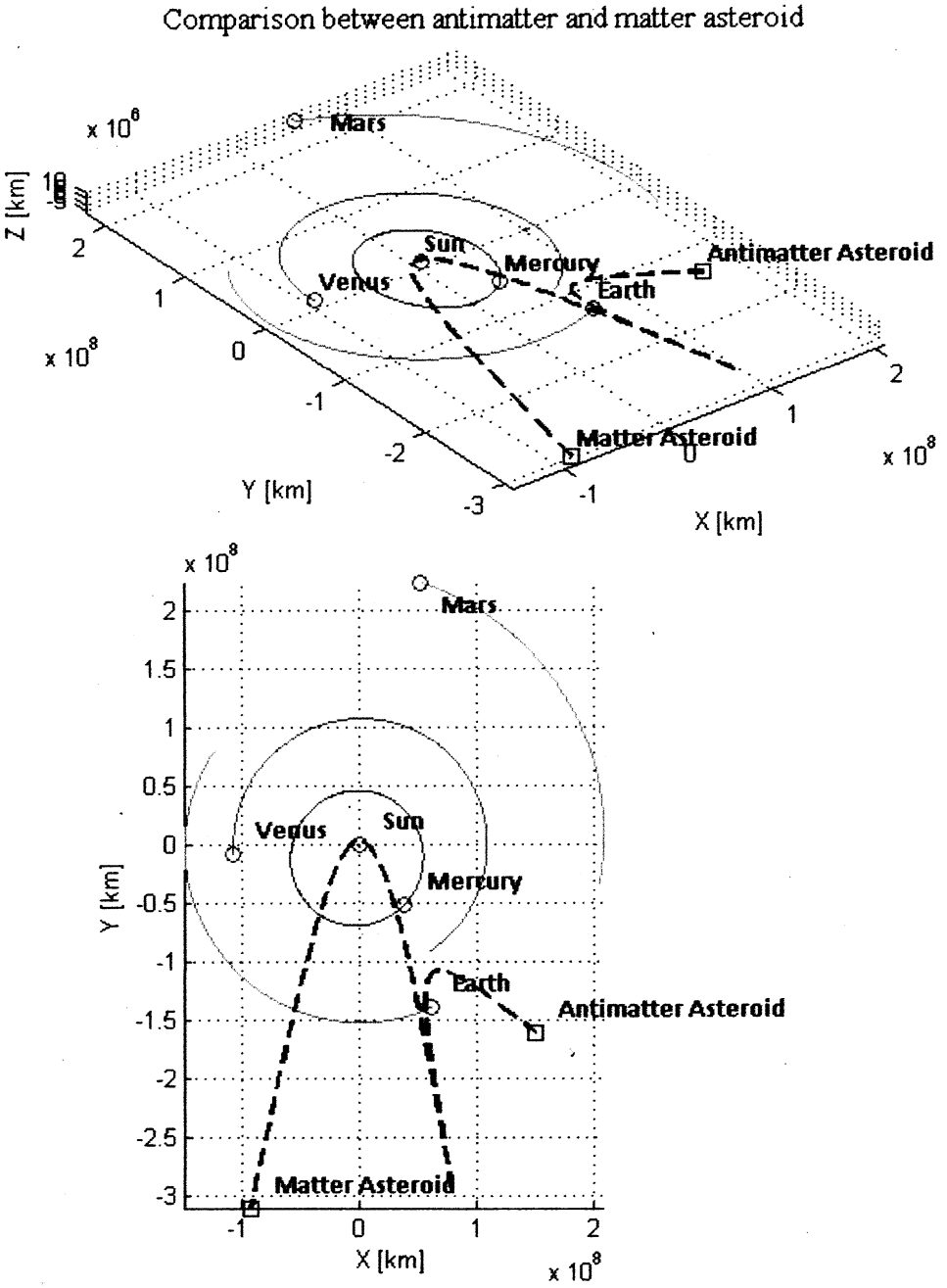


Figure 1 - A comparison between the trajectories of matter and antimatter asteroids starting from the same initial condition

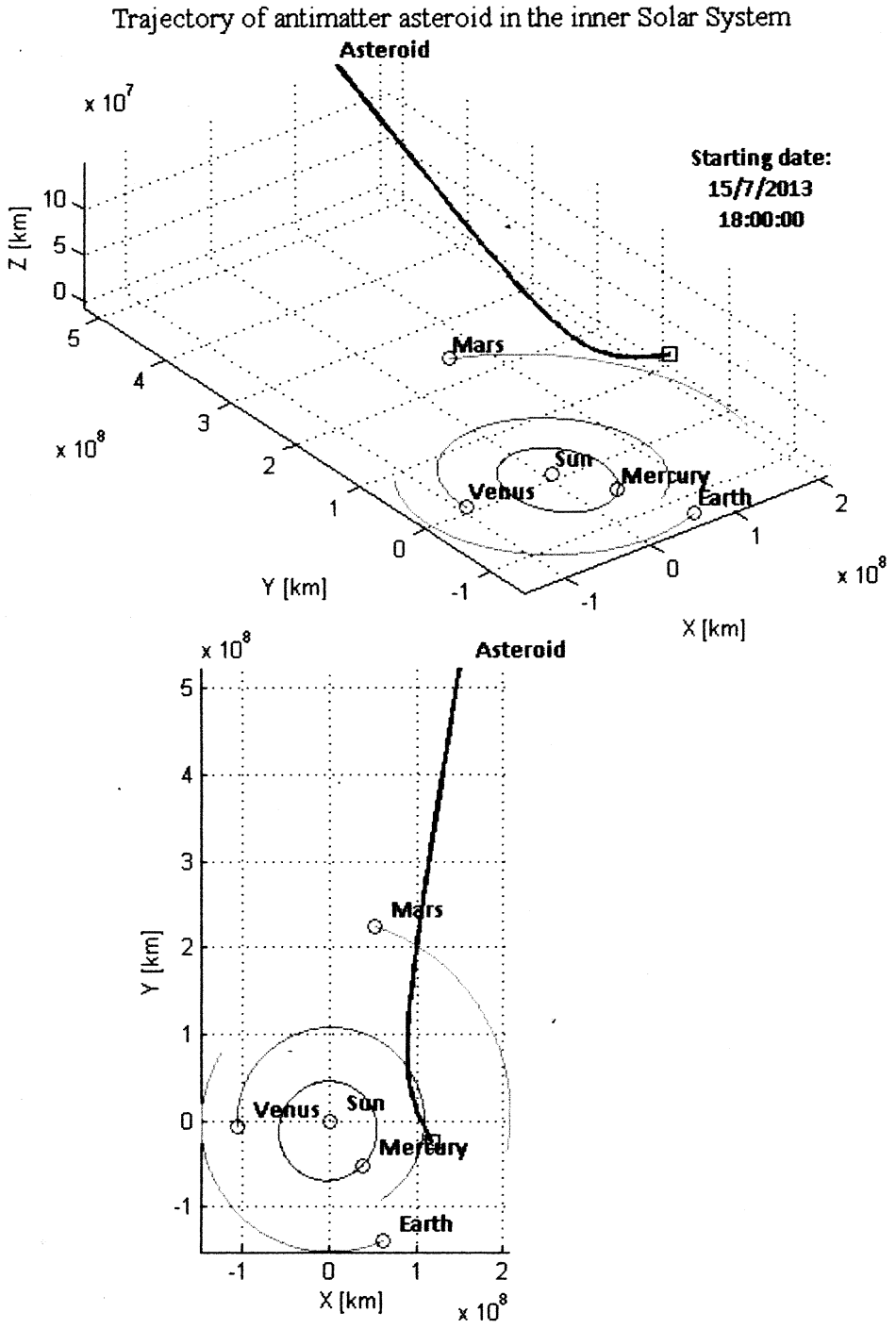


Figure 2 - Antimatter asteroid deflected before crossing the ecliptic plane

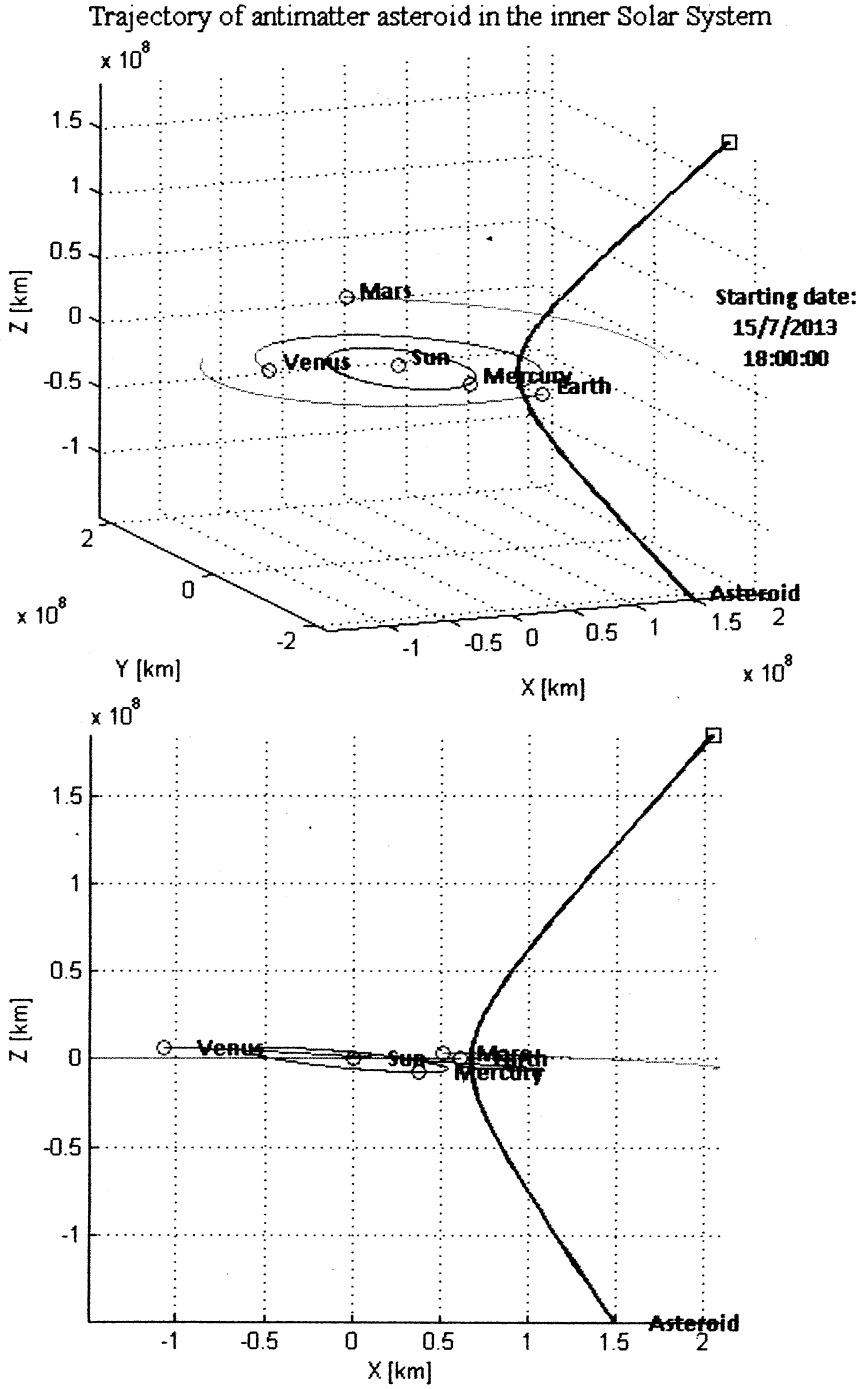


Figure 3 - Antimatter asteroid crossing the ecliptic plane almost perpendicularly

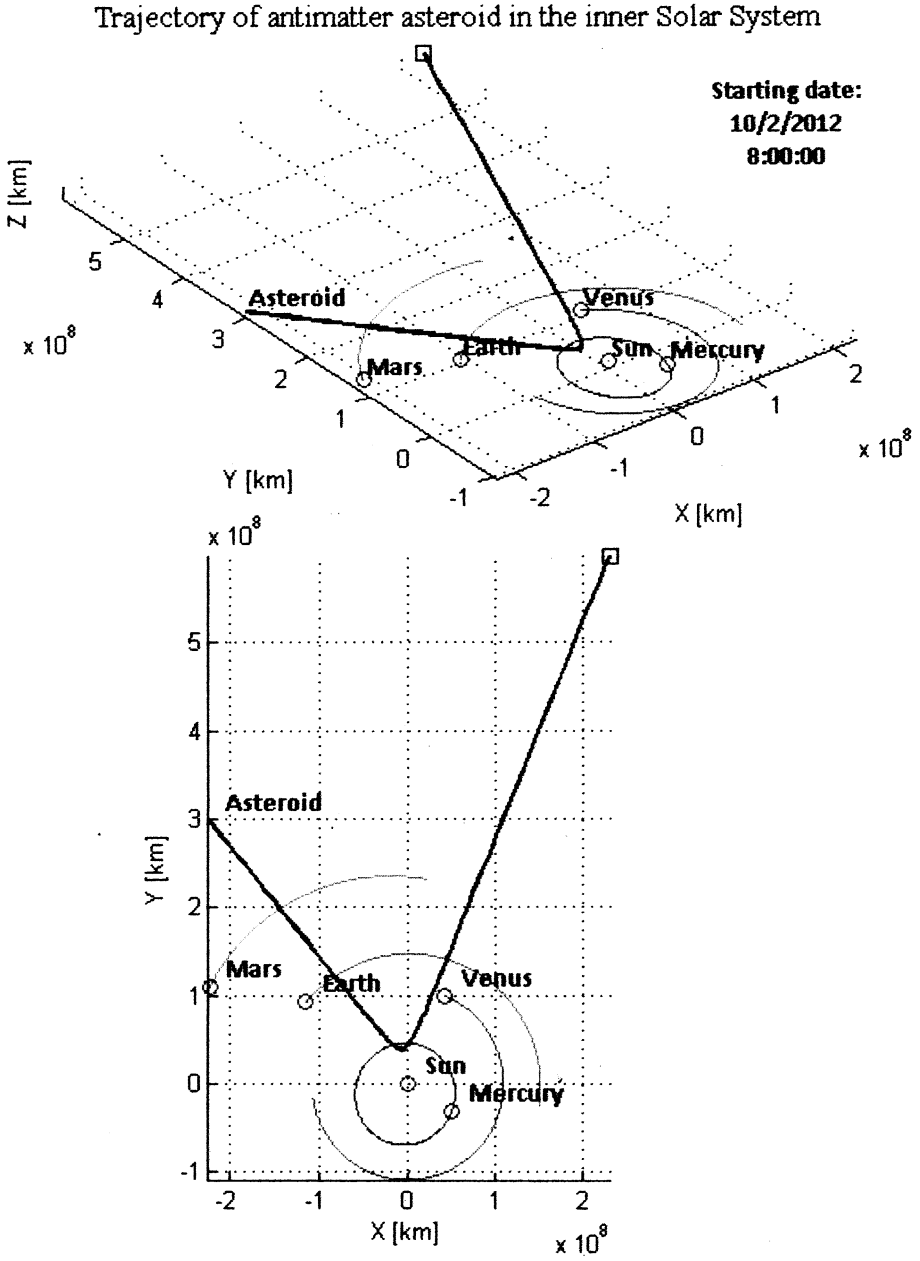


Figure 4 - Antimatter asteroid almost within ecliptic plane deflected by Sun's gravity

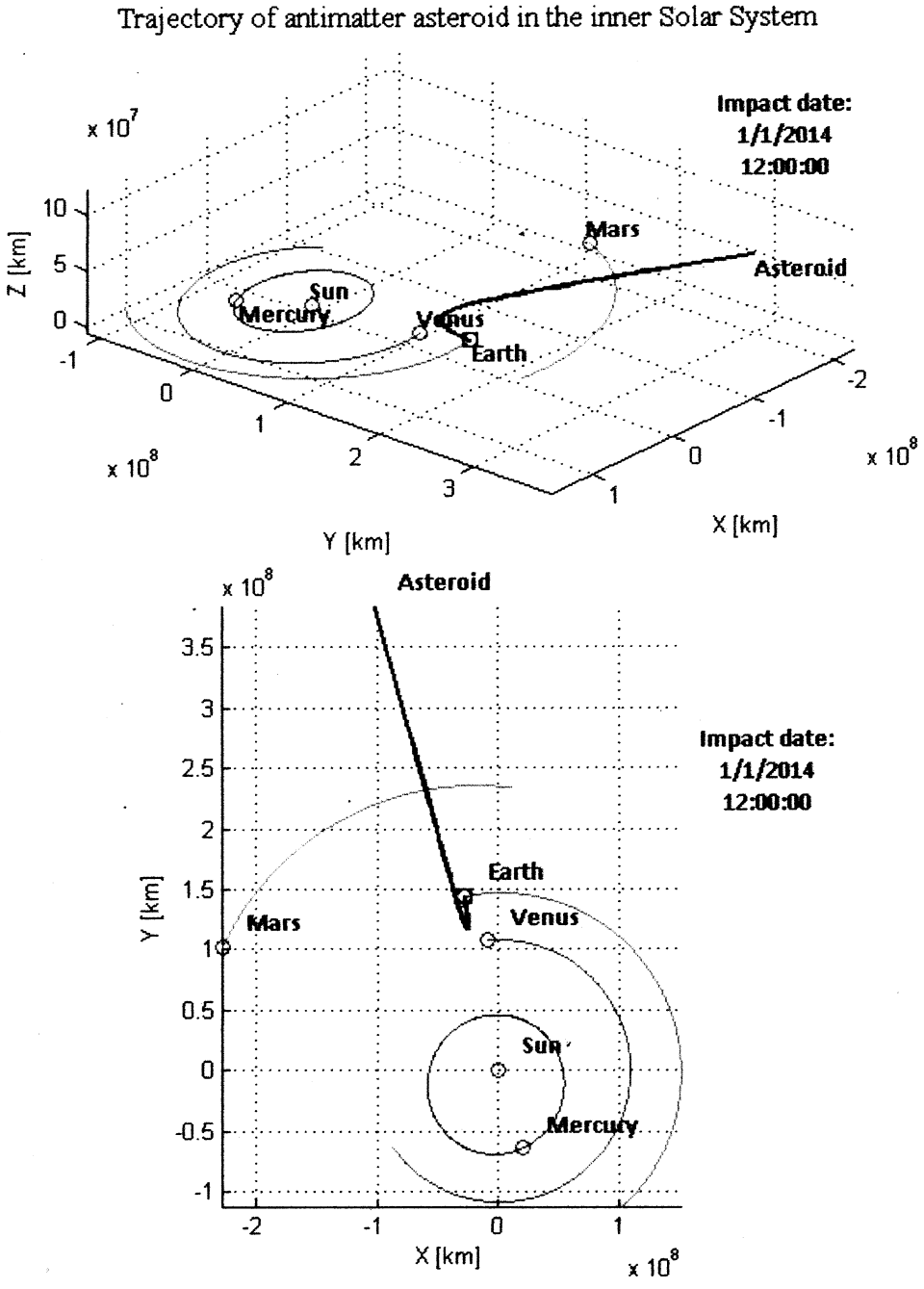


Figure 5 - Antimatter asteroid impacting Earth after having been deflected by Sun's gravity

Trajectory of antimatter asteroid in the inner Solar System

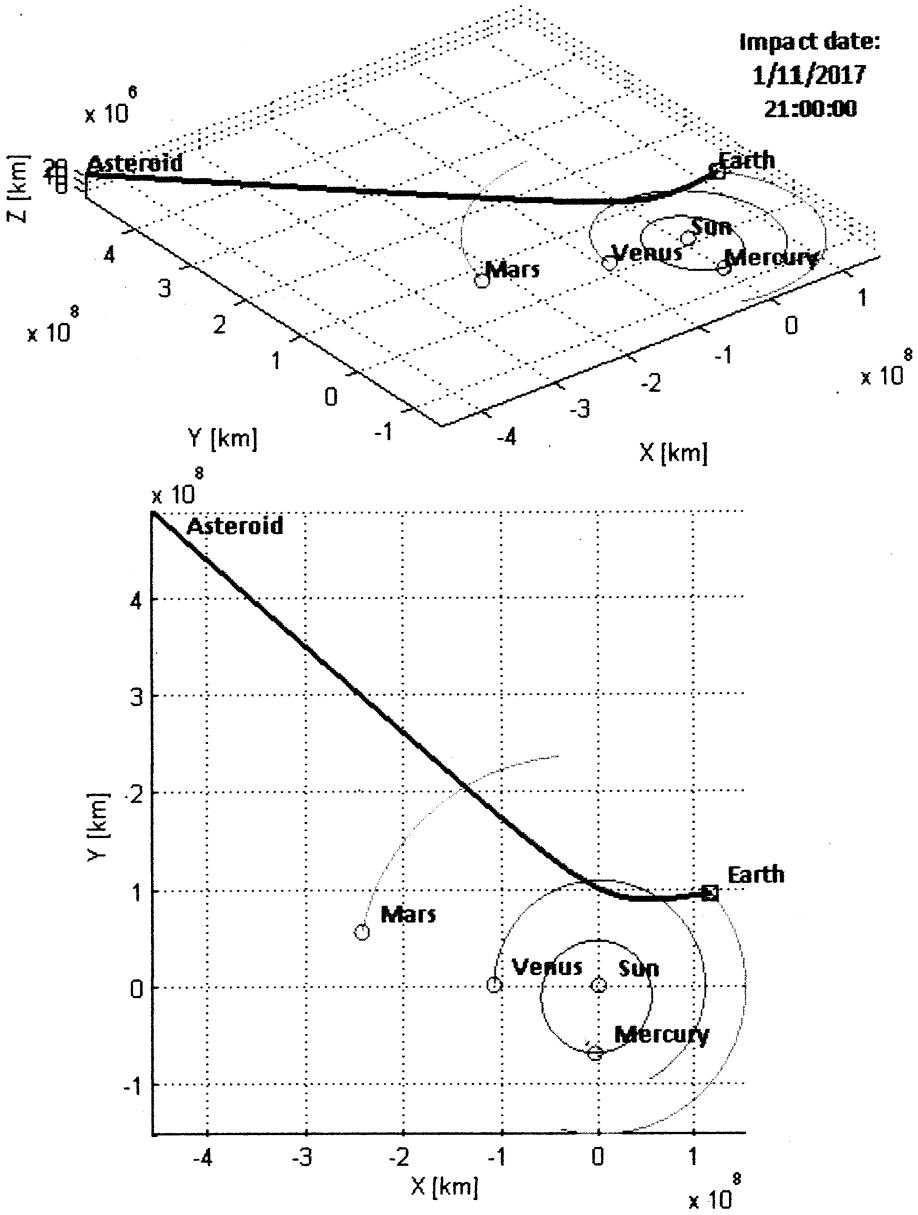


Figure 6 - Antimatter asteroid impacting Earth almost within the ecliptic plane

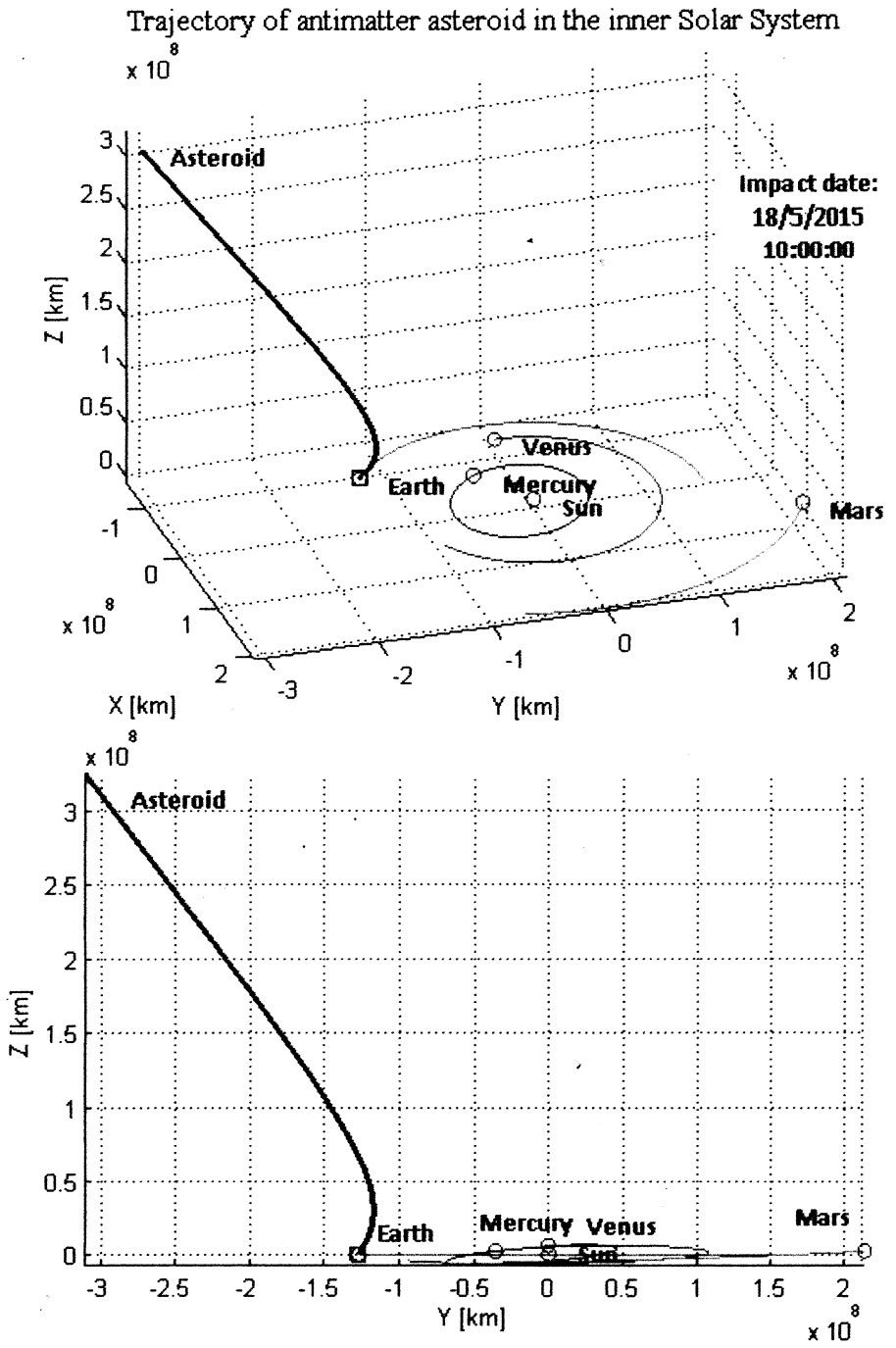


Figure 7 - Antimatter asteroid impacting Earth almost perpendicularly to the ecliptic plane

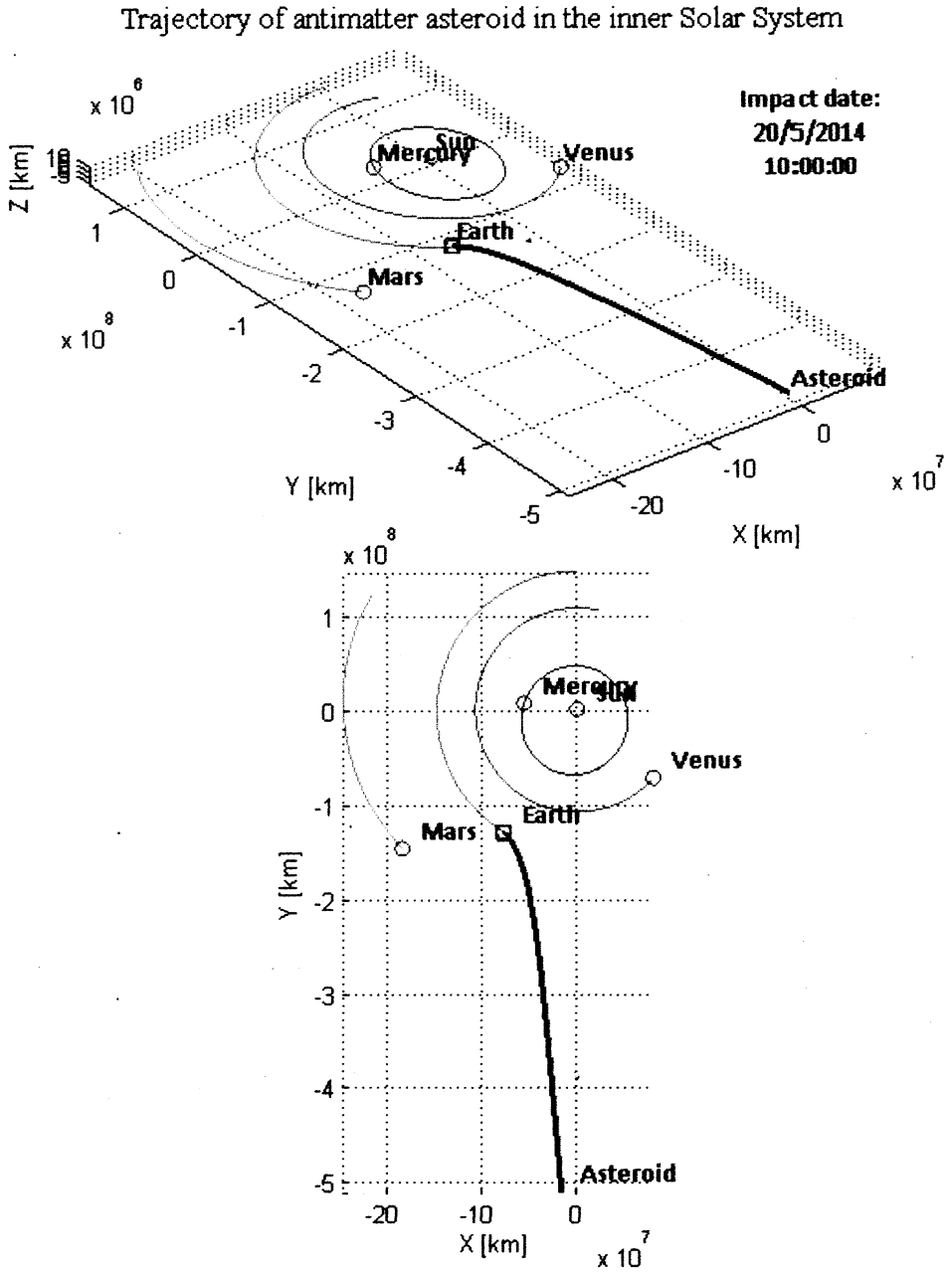


Figure 8 - Antimatter asteroid impacting Earth before being completely deflected by the Sun

4. Considerations

At this point we can make some considerations about these trajectories.
First of all, it is important to look at their shape. We can see from the pictures above that all the trajectories look very similar to hyperbolas.
It is indeed possible to verify that in the case of the two-body problem with an antimatter body and a matter "attractor", the only possible trajectory is the leg of a hyperbola located opposite to the focus with respect to the asymptotes intersection, an orbit which is impossible to obtain in the case of ordinary matter. See Ref. [6] for theoretical background.
If we consider only the Sun-Antimatter asteroid system, the conservation of energy can be written as follows:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu^d}{r} = \frac{v^2}{2} + \frac{\mu}{r} > 0 \qquad (6)$$

Where:
ε is the total energy per unit mass
v is the velocity at an arbitrary point of the trajectory
r is the distance from the center of mass of the Sun
$\mu = GM_{Sun}$ is the gravitational parameter of the Sun

From the above equation we can see that, for a generic matter-antimatter two-body system, total energy is always positive, and this implies the orbit is always open (in the case of elliptic orbits total energy is negative).
Moreover, it is possible to derive the equation of eccentricity, starting from the dynamics of the system:

$$\ddot{\boldsymbol{r}} = \frac{\mu}{r^3}\boldsymbol{r} \qquad (7)$$

Making the cross product of the two members of the equation with the vector $\boldsymbol{h}$, which represents the angular momentum, we have:

$$\ddot{\boldsymbol{r}} \times \boldsymbol{h} = \frac{\mu}{r^3}(\boldsymbol{r} \times \boldsymbol{h}) \qquad (8)$$

After some algebraic manipulations, it is possible to derive the vectorial equation of eccentricity:

$$\boldsymbol{e} = \dot{\boldsymbol{r}} \times \frac{\boldsymbol{h}}{\mu} + \hat{\boldsymbol{r}} \qquad (9)$$

Where:
$\boldsymbol{e}$ is the eccentricity vector
$\hat{\boldsymbol{r}}$ is the unit vector of the vector $\boldsymbol{r}$

From this equation we can derive, with other manipulations, the orbit equation:

$$r = \frac{h^2/\mu}{\mathrm{e}\cos\theta - 1} \qquad (10)$$

At this point we have a representation of our trajectory in its plane, centered in the "attractor" body (the Sun in our case), in terms of radial and angular components.
Now we need to verify if it is still a conic (like in the matter case), and more precisely the kind of conic we have assumed.

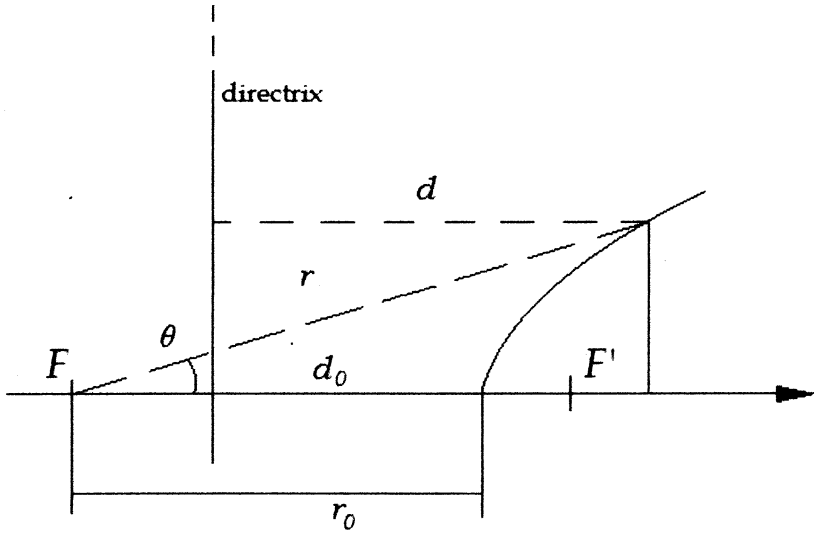


Figure 9 - Representation of a hyperbola according to the geometric definition

One definition of conic is the locus of points whose distances to some point, called a focus, and some line, called a directrix, are in a fixed ratio, called the eccentricity. Therefore we can write (see Figure 9):

$$\frac{r}{d} = \frac{r_0}{d_0} = e \qquad (11)$$

Looking at Figure 9 we can also write (note that the focus considered is the one opposite to the curve with respect to the directrix):

$$r\cos\theta - d = r_0 - d_0 \qquad (12)$$

$$r(e\cos\theta - 1) = r_0(e-1)$$

At this point, with some manipulations:

$$r = \frac{r_0(e-1)}{e\cos\theta - 1} \qquad (13)$$

The equation obtained is in the same form of (10), and considering (9) we can calculate the modulus of eccentricity, verifying it is always bigger than 1 (which corresponds to the hyperbolic case):

$$|e| = \sqrt{\left(\dot{r}\frac{h}{\mu}\right)^2 + \left(\frac{h^2}{\mu r} + 1\right)^2} > 1 \qquad (14)$$

With these calculations we have successfully demonstrated that the generic trajectory of a small antimatter body in the gravitational field of a massive matter body is a conic and more precisely the leg of a hyperbola opposite to the focus with respect to the axis of symmetry (or the intersection of the asymptotes).

In the trajectories above we have the presence of other planets besides the Sun, so we are definitely not in the case of the two-body problem, but still the trajectories look very much like hyperbolas. This is because the gravitational effect of the planets is very small if compared to the effect of the Sun, and negligible outside their spheres of influence.

Another element that is worth investigating is the problem of the identification of a minimum approach speed for an antimatter asteroid that allows an impact with Earth.

In the case of matter asteroids, an important parameter is the kinetic energy at impact, which is connected to the amount of damage caused by the asteroid itself. In the case of antimatter asteroids it is instead more important to know the kinetic energy far from Earth, ideally when entering the Solar System, because, being the interaction repulsive, the kinetic energy is going to decrease while getting closer to the Sun, so a low initial kinetic energy could mean not being able to reach Earth orbit at all. At impact, on the contrary, the energy released depends mostly on the annihilation

between matter and antimatter, so the velocity is less significant (at least in a first approximation).

To have a first idea of the minimum approach speed we can consider again the two-body problem (since the gravity of the Sun is much bigger than the gravity of other planets) and look at the conservation of energy. Calling "0" the initial state (far from Earth) and "f" the final state (impact with Earth), we can write:

$$\frac{v_0^2}{2} + \frac{\mu}{r_0} = \frac{v_f^2}{2} + \frac{\mu}{r_f} \tag{15}$$

$$v_0 = \sqrt{2\left[\frac{v_f^2}{2} + \mu\left(\frac{1}{r_f} - \frac{1}{r_0}\right)\right]} \tag{16}$$

From these equations it is easy to understand that the minimum values of v_0 can be found for the limit case
$v_f = 0$, so when all kinetic energy has been "consumed" and the asteroid reaches Earth orbit with zero velocity.

Considering a mean value for the radius of Earth orbit, we obtain a minimum velocity (at 60 Astronomical Units from the center of mass of the Sun) of 41.77 km/s. A big value for a man-made spacecraft, but not unrealistic for an astronomical object.

In *Figure 10*, we can see a graph showing the minimum velocity needed to impact Earth versus the distance from the Sun at which it is calculated. The graph has an asymptote at 42.12 km/s, which represents the V_∞, the minimum velocity of the asteroid when the distance tends to infinity.

All of this is valid for the two-body case. In reality we have all the planets of the Solar System, including of course Earth, so the trajectories, and hence the minimum velocities, could be slightly different. To have an idea of how much things change with the complete model, we can use our backward propagating model and put the velocity at Earth to zero, cycling only through time, which happens to be the only variable left. It is possible to see the result in *Figure 11*.

As we can see, the minimum velocity has almost periodical variations through time, due to the interactions with the other planets and to the variations in the radius of Earth's orbit throughout the year. But the presence of the planets can also imply that the minimum velocity can be found with impact velocity different than zero. Since the gravitational effect of the planets compared to that of the Sun is almost negligible, we

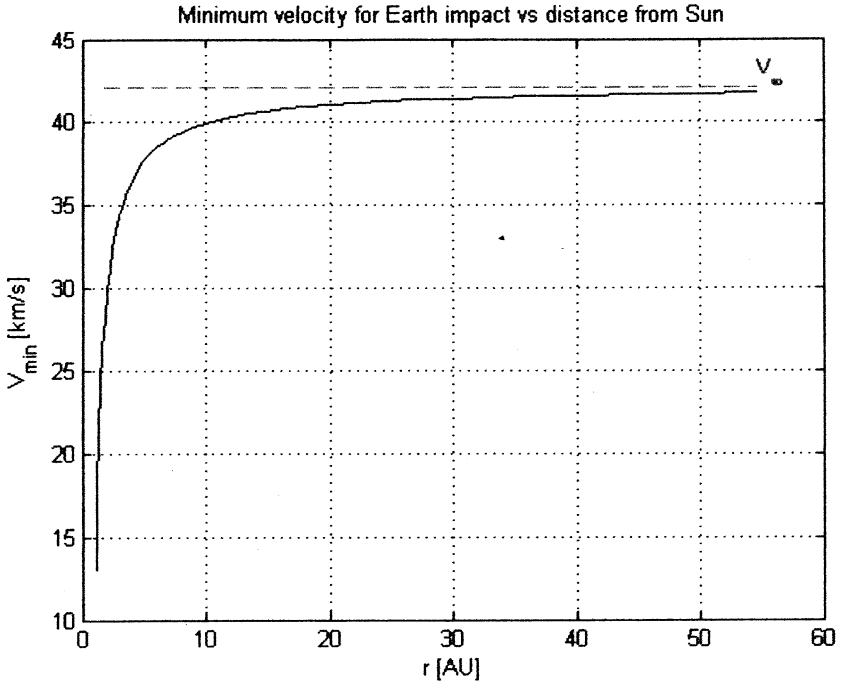


Figure 10 - Minimum velocity needed by an antimatter asteroid to reach Earth calculated at various distances from the Sun using the equations of the 2-body problem and the mean radius of Earth's orbit

do not expect the result to be much different from the two-body case, but there can still be some variations. To verify this hypothesis , we have to check the true trajectories in space that have an impact with Earth to find those with minimum velocity at a given distance from the Sun.

Since a grid search varying all the parameters would be too long and computationally expensive, the best solution is to make an optimization process to find the best trajectory. The three components of velocity at impact and the time of arrival have been used as free parameters, and the modulus of speed has been set at a chosen distance from the Sun as the objective function to be minimized. As a constraint, the velocity has been set to be positive.
The best solution found can be seen in *Figure 12*.

As it is possible to see, the difference from the two-body case is very small, but the solution found is actually better, in terms of low velocity. Many cycles of optimizations have been done varying the initial conditions, since this numerical process is very sensitive to local minima and there is the possibility of missing good solutions because of a wrong initial guess.

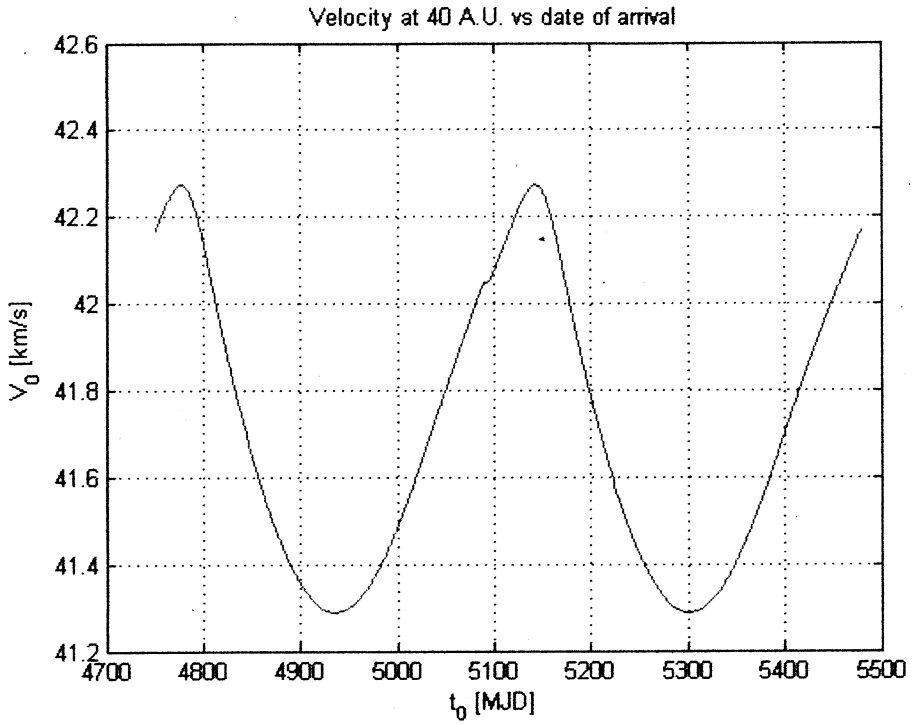


Figure 11 - Minimum velocity needed by an antimatter asteroid to reach Earth at a distance of 40 AU from the Sun, calculated with the integration of dynamical equations of all the major planets, setting final velocity to zero and cycling through time. The time is shown in MJD 2000 (Modified Julian Date, calculated in days starting from 12:00 UTC on 1 Janaury 2000)

Another aspect that can be interesting to investigate is the possible direction of arrival of the asteroid. As we can see, most of figures *Figure* 5 to Figure 9 show trajectories where the impacting antimatter asteroid enters the Solar System in a direction very different from the direction of the Sun. Obviously this depends on the big repulsive action of the Sun itself, that deflects the asteroids too aligned with the Sun-Earth direction away from our planet. We want to verify whether there is a limit in the Sun-Earth-asteroid angle (see *Figure 13*), in order to have collision with Earth. This information could be useful in the future to know where to look more carefully for a possible incoming antimatter asteroid.

Another optimization process has been implemented, using the alpha angle as the objective function to be minimized, and again the three components of velocity at impact and the time of arrival as optimization parameters.

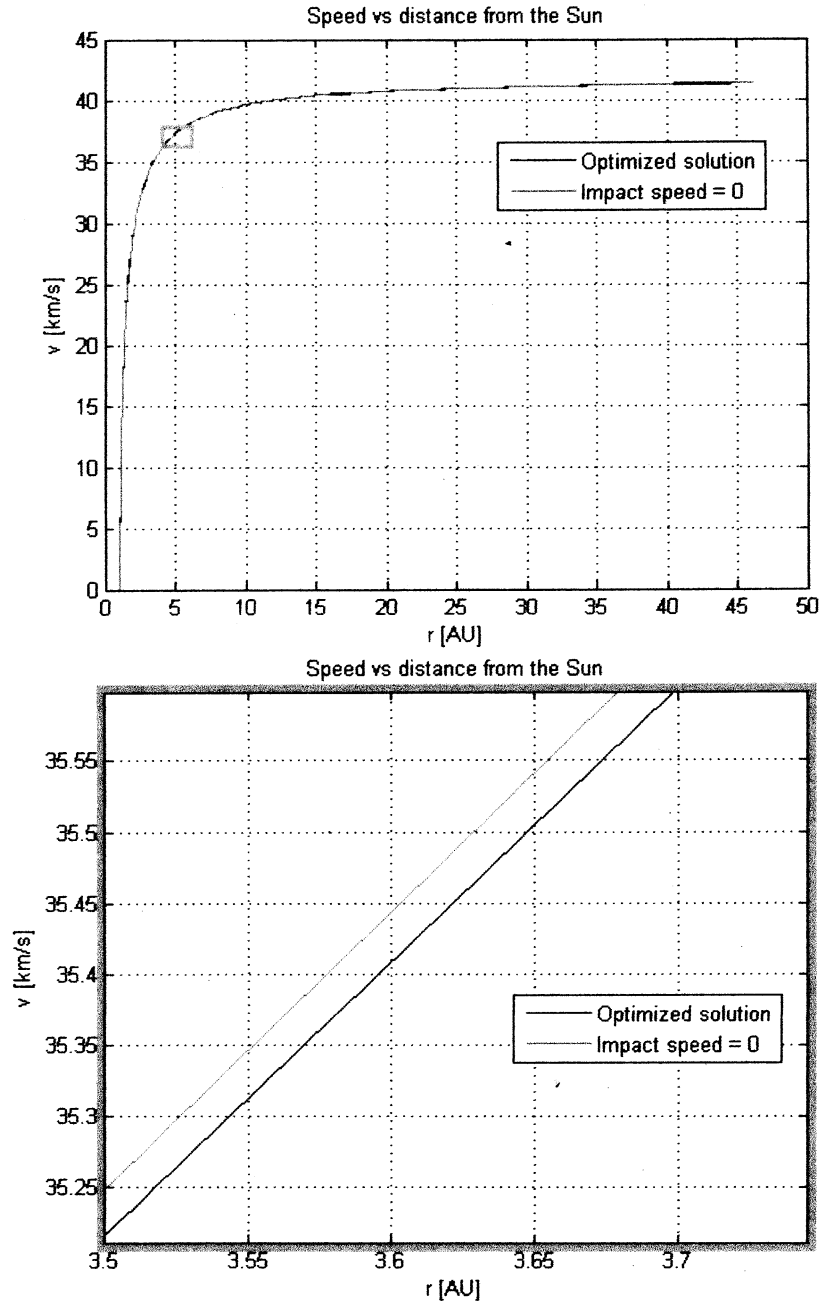


Figure 12 - Best solution found after many cycles of optimizations for the minimum velocity needed by an antimatter asteroid to reach Earth calculated at various distances from the Sun using the complete model. The graph below is a magnification of the graph above in the indicated zone. The red solution is that calculated at the same time of arrival setting the impact speed to zero

Setting no limit to the velocity far from Earth, we obtain a trajectory with a very narrow angle and almost in the shape of a straight line starting very far from the Sun. This is due to the very high velocity of the asteroid (more than 1000 km/s in the example of *Figure 14*), that makes the repulsive effect of the Sun negligible.

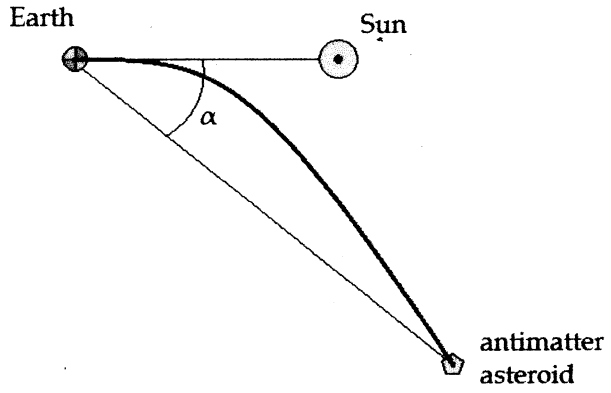


Figure 13 - Schematics of the approach of an antimatter asteroid to our planet, with the indication of the alpha angle between the Earth-Sun vector and the Earth-asteroid vector (using the position of the asteroid at 10 AU from the Sun)

This is kind of a trivial solution, but we can obtain more interesting information by setting a limit to the speed of the asteroid at a given distance from the Sun. By varying this limit it is possible to obtain different values (see *Figure 15*), that represent the minimum alpha angle achievable to impact Earth without exceeding a given velocity at a fixed distance from the Sun (an example is shown in *Figure 16*).

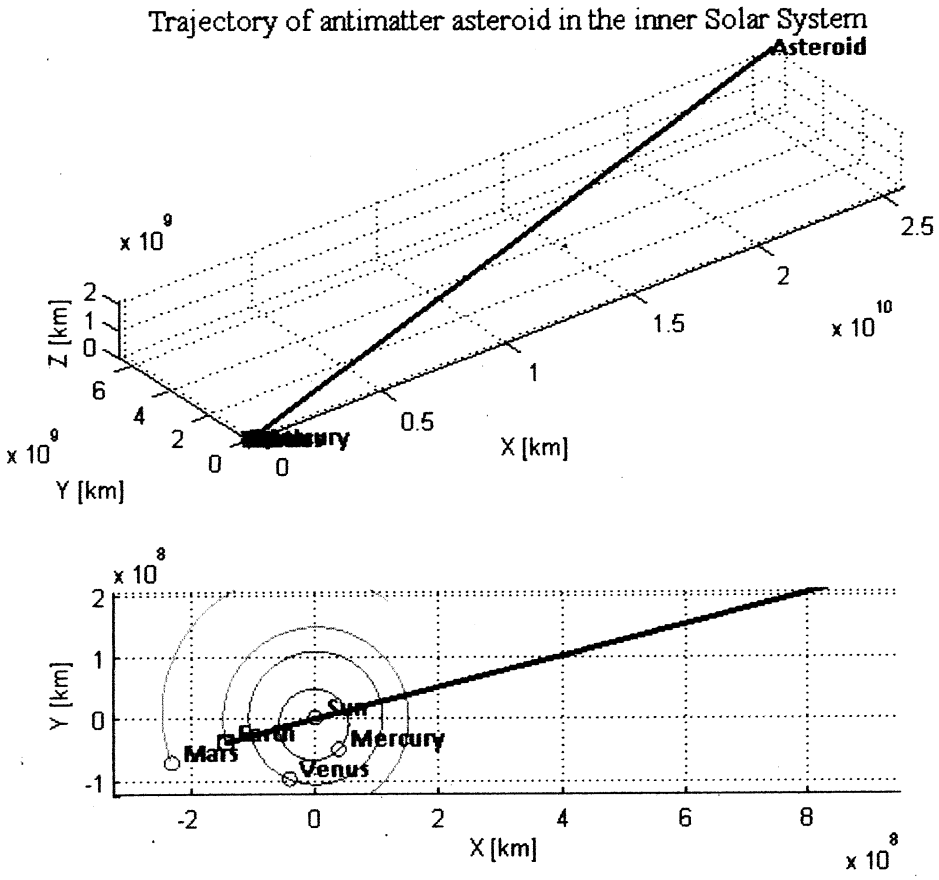


Figure 14 - Trajectory of an antimatter asteroid impacting Earth with as small an alpha angle as possible, calculated with an optimization without constraints on the velocity. In this case the angle obtained is 4.43°, but with a velocity (at 10 AU) of 1037.34 km/sec

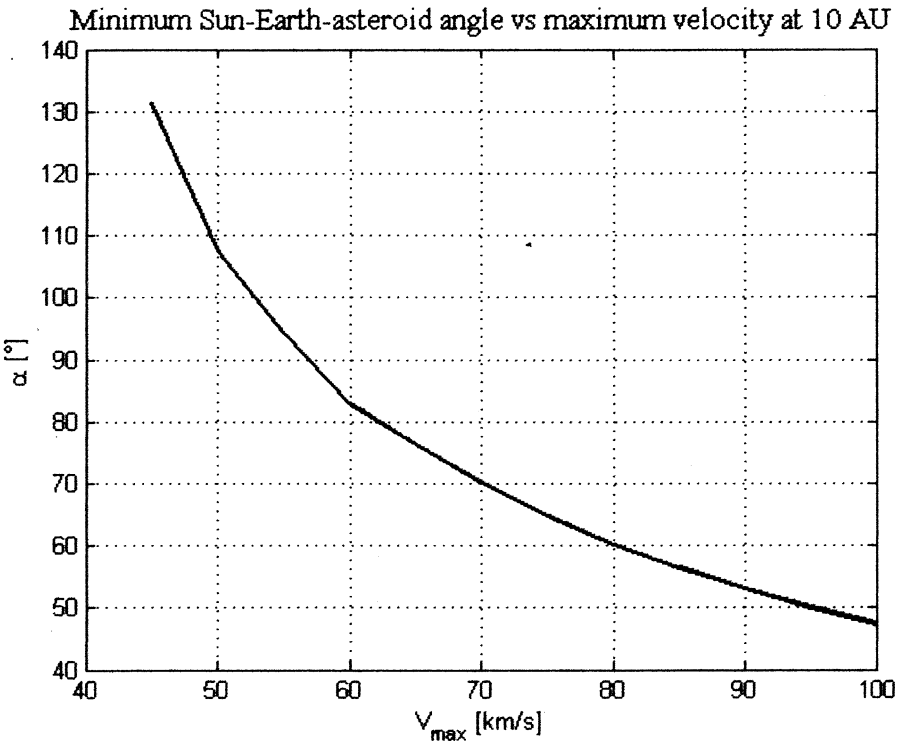


Figure 15 - Minimum alpha angle achievable at different speeds (calculated at 10 AU). This graph has been obtained numerically with different cycles of optimizations, so the values may not belong to the same trajectories, but express a global trend

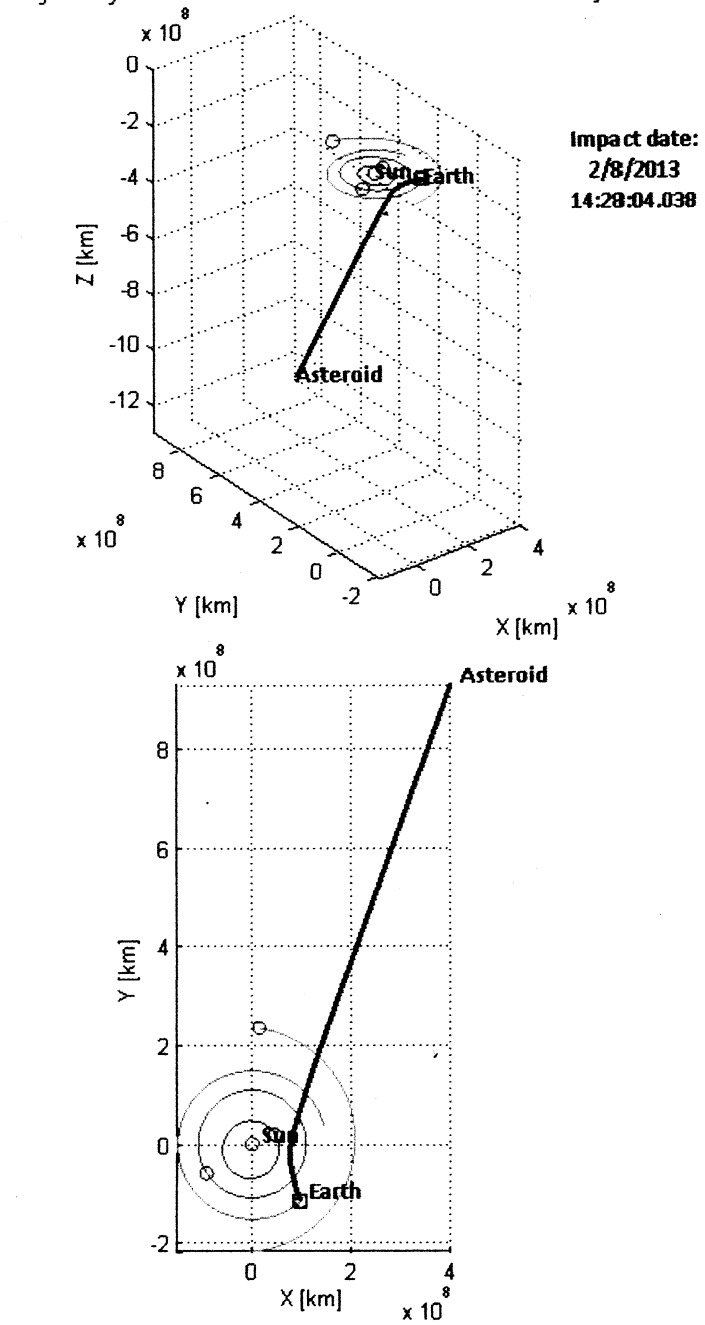


Figure 16 - Example of trajectory with a low alpha angle. Alpha is 69.17°, while the velocity at 10 AU has been limited to 70 km/sec

5. Conclusions

In this paper the author has presented an overview about the problems connected with the trajectories of antimatter bodies entering the Solar System and with the possibilities of impact with Earth. The calculations have been based on Santilli's isodual theory of antimatter [1] because it is the only theory known to the author that has been specifically constructed for the classical representation of neutral antimatter bodies, as needed for antimatter asteroids. In particular, the conjugation from classical and neutral bodies to their antimatter counterparts implies the gravitational repulsion between matter and antimatter which has been assumed at the foundation of these calculations.
The main results obtained can be summarized as follows:

1) An antimatter asteroid can actually enter our Solar System and impact Earth, with initial speeds comparable to those of common celestial bodies

2) An antimatter asteroid would always move around a matter star on an open hyperbolic orbit, with the star itself in the focus opposite to the trajectory.

3) There is a minimum approach speed (roughly 42 km/s) that changes slightly during the year (see *Figure 11*), below which the asteroid would be unable to reach Earth because of the repulsive action of the Sun.

4) Depending on the velocity of the asteroid (see *Figure 15*), there is a minimum angle between the approaching direction of the asteroid and the Sun (as seen from the Earth), below which impact with Earth is not possible.

Of course the model used to achieve these results is not complete, and the results themselves are valid under the correct hypotheses: point-like masses moving in vacuum, asteroid mass negligible with respect to the other masses involved, no interaction other than gravity, so forces are conservative and motion is reversible. These conditions approximate well the phenomena considered in the real world, but there are some effects that have not been considered.

The interaction with solar light has been ignored, in part because its entity is supposed to be secondary with respect to the gravitation, considering also the high speeds involved, and in part because the effect of matter light hitting an antimatter body is still unknown (see [4]).

The effect of the solar wind hitting the asteroid has also been ignored. The solar wind is mainly composed of protons and electrons escaping the Sun's gravity, so at their

impact with the asteroid they would annihilate releasing energy that would probably modify the asteroid's trajectory during time. The entity of this effect would depend mostly on the distance from the Sun and the density and speed of the solar wind itself, which is very variable during time.
Another effect ignored in this study is that of the asteroid belt, located between Mars and Jupiter orbits, that again could slightly modify the trajectory, but this effect is supposed to be very small if compared to the gravitation of the Sun and other planets.

Further studies are needed to better understand the behavior of these bodies, since the research in this field is only at the beginning. Future developments should focus on:

- the effect of matter light on antimatter, not only for the consequences on the trajectories, but also for the detection itself of the antimatter asteroids (as explained in [4]) without which we wouldn't be able to locate an incoming asteroid. Important steps forward have been made for the detection of antimatter stars and galaxies (see [2] and [3]), but we still don't know anything about cold bodies that don't have emissions of their own;

- the effect of the solar wind on the trajectories of antimatter bodies. Although weak, especially far from the Sun, for an approaching asteroid this effect could become more significant with the passage of time. Moreover, the annihilation of electrons and protons with the asteroid's surface would produce both matter and antimatter radiations, and the former, if intense enough, could be seen from Earth and contribute to the detection of the asteroid itself, so this aspect could be particularly important to investigate on.

- the effects of the entrance into the atmosphere, that would be important of course to determine the behavior of the antimatter body and the damages it could deliver to our planet.

Acknowledgements

The author would like to thank the referees of the Hadronic Journal for penetrating criticisms and constructive suggestions that have been invaluable for the completion of this paper. Additional thanks are due to the R.M. Santilli Foundation for the financial support.

References

[1] R. M. Santilli, "Isodual Theory of Antimatter with Application to Antigravity, Grand Unifcation and Cosmology", Springer, 2006.

[2] R. M. Santilli, ""Apparent detection of antimatter galaxies via a refractive telescope with concave lenses"," *Clifford Analysis, Clifford Algebras and their Applications (Cambridge, UK),* vol. 3, pp. 1-26, 2014.

[3] P. M. Bhujbal, J. V. Kadeisvili, A. Nas, S. Randall and T. R. Shelke, ""Preliminary confirmation of antimatter detection via Santilli's telescope with concave lenses"," *Clifford Analysis, Clifford Algebras and their Applications (Cambridge, UK),* vol. 3, pp. 27-39, 2014.

[4] R. Anderson, A. A. Bhalekar, B. Davvaz and P. Muktibodh, ""An introduction to Santilli's isodual representation of antimatter and the ensuing problem of detecting antimatter asteroids"," *Numta Bulletin,* no. 67, pp. 1-33, 2013.

[5] S. Beghella Bartoli, "Trajectory Optimization for ESMO Mission", Milan: Politecnico di Milano, 2012.

[6] D. A. Vallado, "Fundamentals of Astrodynamics and Applications", El Segundo : Space Technology Library: Microcosm Press, 2001.

THE MATRIX FIELD THEORY AND THE GENERALIZED SCHWARZSCHILD SOLUTION OF THE METRIC MATRIX EQUATION

Alexander D. Dymnikov
Louisiana Accelerator Center/Physics Dept/
Louisiana University at Lafayette
320 Cajundome Blvd., Lafayette, LA 70506 USA
dymnikov@lousiana.edu
addymnikov@gmail.com

Received March 11, 2014
Revised April 8, 2014

Abstract

The generalized Schwarzschild solution of the metric matrix equation is given, using the novel, pure mathematical, matrix theory of the field in *n*-dimensional metric space[1], where the physical results are the interpretations of the mathematical results. The short content of this theory is given.
The obtained equations of motion in these metric spaces describe the motion of the mass m in the field of the concentrated mass M and the mass density.

Keywords: Matrix field theory; Schwarzschild solution; Metric matrix; Metric matrix equations; Matrix equations of field; Eigenvectors; Contravariant and covariant vectors; Concentrated mass; Mass density.

Copyright © 2014 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

THE SHORT CONTENT OF THE MATRIX FIELD THEORY

PART 1. N–DIMENSIONAL METRIC SPACE

1.VECTOR AND MATRIX FUNCTIONS, DIFFERENTIAL OPERATORS AND THE EQUATIONS OF THE FIELD

1.1 Coordinate vectors *x* and *Q*.

The mathematical point is described by the set of *n* parameters (or coordinates). The position (the set of *n* parameters) of any mathematical point is described by an *n*-coordinate vectors *x* or by *n*-coordinate vector function Q(*x*), where column vectors are written as

$$x \equiv \{\{x_1\},\{x_2\},...,\{x_n\}\} \equiv \begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix},$$

$$Q \equiv \{\{Q_1(x)\},\{Q_2(x)\},...,\{Q_n(x)\}\} \equiv \begin{pmatrix} Q_1(x) \\ Q_2(x) \\ ... \\ Q_n(x) \end{pmatrix}$$

and transposed (row) vectors $\widetilde{x}$ and $\widetilde{Q}$ as

$$\widetilde{x} \equiv \{\{x_1, x_2, ...x_n\}\} \equiv (x_1, x_2, ..., x_n),$$

$$\widetilde{Q} \equiv \{\{Q_1(x), Q_2(x), ...Q_n(x)\}\} \equiv (Q_1(x), Q_2(x), ..., Q_n(x))$$

1.2 Differential vector- operators $\partial(x)$ and $\partial(Q)$.

For the *n*-vectors {*x*} and {*Q*} the differential *n*-vector-operators $\{\partial(x)\}$ and $\{\partial(Q)\}$
are written as the column vectors $\partial(x)$ and $\partial(Q)$

$$\partial(x) \equiv \{\{\partial_1(x)\}, ..., \{\partial_n(x)\}\} = \begin{pmatrix} \partial_1(x) \\ ... \\ \partial_n(x) \end{pmatrix},$$

$$\partial(Q) \equiv \{\{\partial_1(Q)\}, ..., \{\partial_n(Q)\}\} = \begin{pmatrix} \partial_1(Q) \\ ... \\ \partial_n(Q) \end{pmatrix}$$

or as the row vectors $\tilde{\partial}(x)$ and $\tilde{\partial}(Q)$,

$$\tilde{\partial}(x) \equiv \{\{\partial_1(x), ..., \partial_n(x)\}\}) \qquad \tilde{\partial}(Q) \equiv \{\{\partial_1(Q), ..., \partial_n(Q)\}\})$$

where

$$\partial_i(x) \equiv \frac{\partial}{\partial x_i}, \qquad \partial_i(Q) \equiv \frac{\partial}{\partial Q_i}.$$

1.3 Scalar differential operator and exact differential.

The scalar differential operator *d*, the exact differentials *dQ* and the frame matrix *e* are defined by the following way:

$d = \tilde{\partial}(x)\, dx = \tilde{\partial}(Q)\, dQ \qquad dQ = Q\, d = Q\, \tilde{\partial}(x)\, dx = e\, dx, \quad e \equiv Q\, \tilde{\partial}(x)$

$e_{ik} = \partial Q_i / \partial x_k$.

From the expressions for *dQ* and *dx* it follows

$$dQ = e\, dx = e\, xd = e\, x\, \tilde{\partial}(Q)\, dQ, \qquad x\, \tilde{\partial}(Q) = e^{-1}$$

1.4 Short operator notations.

In many cases we will use the short notations,

$$\partial \equiv \partial(x), \quad \partial_k \equiv \partial_k(x) = \frac{\partial}{\partial x_k}, \quad \partial(Q) \equiv D, \quad D_k \equiv \partial_k(Q) = \frac{\partial}{\partial Q_k}$$

$$d \equiv \tilde{\partial} dx = d\tilde{x} \partial = \frac{\partial}{\partial x_\beta} dx_\beta, \quad d \equiv \tilde{D}\, dQ = d\tilde{Q}\, D = \frac{\partial}{\partial Q_\beta} dQ_\beta.$$

The summation is understood to be carried out over any *repeated* Greek index symbol that appears twice in any expression.

1.5 The absolute coordinate vector, the contravariant and covariant vectors.
The non-integrable but differentiable frame matrix e can be written in the form

$$e = \Omega e_0, \quad \tilde{\Omega}\Omega = I_n$$

where e_0 is an integrable frame matrix and Ω is a non-integrable but differentiable orthogonal matrix. The matrix form of the infinitesimal invariant interval is given by

$$d\tau^2 = d\tilde{x}\, g(x)\, dx = d\tilde{x}\, \tilde{e}\, e\, dx = d\tilde{x}\, \tilde{e}_0\, e_0\, dx = d\tilde{Q}\, dQ,$$

with

$$g(x) = \tilde{e}\, e = \tilde{e}_0\, e_0,$$

where e is –a non-integrable but differentiable frame matrix, g(x) is an integrable metric matrix in the coordinate space x, $d\tau$ is the infinitesimal invariant interval. We call the coordinate vector Q as the *absolute* (invariant) coordinate vector and the frame matrix e as the *absolute* (invariant) frame matrix.

Any $n \times n$ matrix function F is integrable, if

$$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) F = 0, \quad \alpha, \beta = 1, \ldots, n.$$

Any $n \times n$ matrix function F is non-integrable, if

$$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) F \neq 0, \quad \alpha, \beta = 1, \ldots, n.$$

The vectors dQ and dx are the **contravariant** vectors and the vectors $\partial(x)$ and $\partial(Q)$ are the **covariant** vectors. The relation

$$dQ = e\, dx,$$

links the **contravariant** coordinate vector dQ and the **contravariant** coordinate vector dx. The relation

$$\partial(Q) = \tilde{e}^{-1}\, \partial(x)$$

connects the **covariant** coordinate vector operator $\partial(Q)$ and the **covariant** vector operator $\partial(x)$.

In the matrix theory the indices are usually not shown. If it is necessary the following notations for matrix or vector indices are used: $\underline{k}$ – covariant index,

$\bar{k}$ - contravariant index. The matrix $e = Q\tilde{\partial}(x)$ is a contravariant-covariant matrix.

1.6 Four coordinate forms of any absolute matrix *P*.

We will use four coordinate forms of any absolute matrix *P*: the twice contravariant matrix $\bar{P}$, the twice covariant matrix $\underline{P}$, the covariant-contravariant matrix $\underset{\rightarrow}{P}$ and the contravariant-covariant matrix $\vec{P}$, where

$$\bar{P} \equiv e^{-1} P \tilde{e}^{-1}, \quad \underline{P} \equiv \tilde{e} P e \quad \underset{\rightarrow}{P} \equiv \tilde{e} P \tilde{e}^{-1} = g \bar{P} = \underline{P} g^{-1}$$

$$\vec{P} \equiv e^{-1} P e = \bar{P} g = g^{-1} \underline{P}$$

1.7 Unit vectors and matrices.

We introduce the contravariant unit vector $\bar{i}(\lambda) \equiv \partial_\lambda x$, covariant unit vector $\underline{i}(\lambda) \equiv \partial x_\lambda$, and the unit vector $i(\lambda)$, where $i_j(\lambda) = \delta_{j\lambda}$. If we are not interested in the nature of the unit vector, we use the unit vector $i(\lambda)$. As an example, for *n* = 4,

$$i(1) = \{1,0,0,0\}, \quad i(2) = \{0,1,0,0\}, \quad i(3) = \{0,0,1,0\}, \quad i(4) = \{0,0,0,1\}$$

We use the following definition of the unit (or identity) matrix:

$$I_n = i(\mu)\tilde{i}(\mu) = i(1)\tilde{i}(1) + i(2)\tilde{i}(2) + \ldots + i(n)\tilde{i}(n).$$

As an example,

$$I_4 \equiv \{\{1, 0, 0, 0\}, \{0,1,0,0\}, \{0,0,1,0\}, \{0,0,0,1\}\}$$

1.8 The compatibility conditions for the vector *Q*.

In order that the differential *dQ* be exact differential, it is necessary and sufficient that there are the relations:

$$\partial_k(x)\partial_j(x)Q - \partial_j(x)\partial_k(x)Q = \partial_k e i(j) - \partial_j e i(k) = 0, \quad k, j = 1,\ldots,n$$

The equations

$$\partial_k e i(j) - \partial_j e i(k) = 0, \quad k, j = 1,\ldots,n$$

are called **the compatibility conditions** for the vector *Q*.

1.9 The antisymmetric matrix-differential operators.

The antisymmetric matrices - differential operators $[\partial\tilde{\partial}]$ and $[D\tilde{D}]$,

$$[\partial\tilde{\partial}] \equiv \{\{[\partial\tilde{\partial}]_{11}, [\partial\tilde{\partial}]_{12}, \ldots, [\partial\tilde{\partial}]_{1n}\}, \ldots, \{[\partial\tilde{\partial}]_{n1}, [\partial\tilde{\partial}]_{n2}, \ldots, [\partial\tilde{\partial}]_{nn}\}\},$$

$$[D\tilde{D}] \equiv \{\{[D\tilde{D}]_{11}, [D\tilde{D}]_{12}, \ldots, [D\tilde{D}]_{1n}\}, \ldots, \{[D\tilde{D}]_{n1}, [D\tilde{D}]_{n2}, \ldots, [D\tilde{D}]_{nn}\}\},$$

are called **the compatibility matrix – operators** and are defined by the elements

$$[\partial\tilde{\partial}]_{ij} \equiv \partial_i\partial_j - \partial_j\partial_i, \quad [D\tilde{D}]_{ij} \equiv D_iD_j - D_jD_i, \quad \partial_k = \frac{\partial}{\partial x_k}, \quad D_k = \frac{\partial}{\partial Q_k}.$$

The compatibility matrix - operators may be written in the form

$$[\partial\tilde{\partial}] = [\partial\tilde{\partial}]_{\alpha\beta}\, i(\alpha)\tilde{i}(\beta), \quad [D\tilde{D}] = [D\tilde{D}]_{\alpha\beta}\, i(\alpha)\tilde{i}(\beta).$$

The expression

$$[\partial\tilde{\partial}] = \tilde{e}[D\tilde{D}]e$$

is the transformation from the *absolute* twice covariant differential operator matrix $[D\tilde{D}]$ to the twice covariant coordinate differential operator matrix $[\partial\tilde{\partial}]$.

2. DERIVATIVES OF THE NON-INTEGRABLE BUT DIFFERENTIABLE FRAME MATRIX.

2.1 One-index matrices.

Let us consider the one-index matrix function h^c, where

$$h^c \equiv \tilde{e}\,\partial_c e, \quad c = 1, 2, \ldots, n, \quad h^c_{ab} = \tilde{i}(a)\tilde{e}\,\partial_c\, e\, i(b),$$

and e is the matrix function. From the compatibility conditions

$$\boxed{\partial_j\, e\, i(k) = \partial_k\, e\, i(j), \quad j, k = 1, \ldots, n},$$

we obtain the following relations between the elements of the matrix h^c:

$$h^c_{ab} = \tilde{i}(a)\tilde{e}\,\partial_c\, e i(b) = \tilde{i}(a)\tilde{e}\,\partial_b\, e i(c) = h^b_{ac},$$

$$h^a_{bc} = \tilde{i}(b)\tilde{e}\,\partial_a\, e i(c) = \tilde{i}(b)\tilde{e}\,\partial_c\, e i(a) = h^c_{ba},$$

$$h^a_{cb} = \tilde{i}(c)\tilde{e}\,\partial_a\, e i(b) = \tilde{i}(c)\tilde{e}\,\partial_b\, e i(a) = h^b_{ca}.$$

It is possible to present the matrix function h^c as the sum of two matrix functions with different symmetry

$$h^c = \gamma^c + \lambda^c,$$

with

$$\lambda^c_{ab} \equiv \frac{1}{2}(h^c_{ab} - h^b_{ac} + h^a_{cb} - h^b_{ca} + h^a_{bc} - h^c_{ba}),$$

$$\gamma^c_{ab} \equiv \frac{1}{2}(h^c_{ab} + h^b_{ac} + h^b_{ca} - h^a_{bc} - h^a_{cb} + h^c_{ba}),$$

$$\boxed{\gamma^c_{ab} = \gamma^b_{ac}}, \qquad \boxed{\lambda^c = -\widetilde{\lambda}^c, \quad \lambda^c_{ab} = -\lambda^c_{ba}}.$$

Here γ^c_{ab} has symmetry in c and b indices, and matrix λ^c is an antisymmetric matrix.

From the equations of compatibility conditions

$$h^a_{bc} = h^c_{ba}, \quad h^c_{ab} = h^b_{ac}, \quad h^a_{cb} = h^b_{ca}$$

it follows, that the matrix λ^c is zero matrix. Therefore

$$h^c = \gamma^c, \quad \gamma^c_{ab} \equiv \frac{1}{2}(h^c_{ab} + h^b_{ac} + h^b_{ca} - h^a_{bc} - h^a_{cb} + h^c_{ba}),$$

or

$$(\widetilde{e}\,\partial_c e)_{ab} = \gamma^c_{ab} = \frac{1}{2}((h^c_{ab} + h^c_{ba}) + (h^b_{ca} + h^b_{ac}) - (h^a_{bc} + h^a_{cb})) \quad c = 1, 2, \ldots, n,$$

From the last equation and from the equations,

$$\widetilde{e}\, e = g(x),$$

$$\partial_c g_{ab} = h^c_{ab} + h^c_{ba}, \quad \partial_b g_{ac} = h^b_{ac} + h^b_{ca}, \quad \partial_a g_{bc} = h^a_{bc} + h^a_{cb},$$

it follows,

$$\gamma^c = \frac{1}{2}(\partial_c g + g\, i(c)\widetilde{\partial} - \partial \widetilde{i}(c)\, g),$$

$$\gamma^c_{ab} = \frac{1}{2}(\partial_c g_{ab} + \partial_b g_{ac} - \partial_a g_{bc}) = \Gamma_{a,cb}$$

2.2 The one-index Christoffel matrices of the first kind and of the second kind.

In the above equations the matrix elements γ^c_{ab} are the *Christoffel symbols* $\Gamma_{a,cb}$ *of the first kind,* which are used in the general theory of relativity. We call this matrix γ^c as the *one-index Christoffel matrix of the first kind*. We call the matrix σ^m, where

$$\partial_m e = \tilde{e}^{-1}\gamma^m \doteq e\sigma^m,$$

$$\sigma^m \equiv e^{-1}\partial_m e = g^{-1}\gamma^m,$$

$$\sigma^{\alpha}_{\mu\beta} = \frac{1}{2} g^{-1}_{\mu\rho}(\partial_\beta g_{\alpha\rho} + \partial_\alpha g_{\rho\beta} - \partial_\rho g_{\alpha\beta}) = \Gamma^{\mu}_{\alpha\beta}$$

as the one-index *Christoffel matrix of the second kind*. The matrices γ^m and σ^m have the following properties

$$\gamma^m i(k) = \gamma^k i(m), \quad \sigma^m i(k) = \sigma^k i(m).$$

2.3 The second derivatives of the non-integrable but differentiable matrices *e*.

Using the first derivatives it is easy to obtain the second derivatives of the non-integrable but differentiable matrices e and e^{-1},

$$\partial_a\partial_b e = e(\partial_a\sigma^b + \sigma^a\sigma^b), \qquad \partial_a\partial_b e^{-1} = (-\partial_a\sigma^b + \sigma^b\sigma^a)e^{-1}$$

$$a = 1,\ldots,n-1; \quad b = 2,\ldots,n; \quad b > a$$

Acting by the differential compatibility operator $\left[\partial\tilde{\partial}\right]_{ab}$ on the frame matrix e, we obtain

$$[\partial\tilde{\partial}]_{ab} e = e\sigma^{ab}, \; \sigma^{ab}_{mk} = \sigma^{\underline{a}\underline{b}}_{\underline{m}\underline{k}},$$

where the two-index matrix σ^{ab} is defined by expression,

$$\sigma^{ab} \equiv \partial_a\sigma^b - \partial_b\sigma^a + \sigma^a\sigma^b - \sigma^b\sigma^a = e^{-1}[\partial\tilde{\partial}]_{ab,}e,$$

$$a = 1,\ldots,n-1; \quad b = 2,\ldots,n; \quad b > a$$

We call the two-index matrix σ^{ab} as **the two-index curvature matrix**. The elements σ^{ab}_{mk} of the two-index curvature matrix σ^{ab} are the elements of the Riemann curvature tensor R^m_{kab},

$$\sigma^{ab}_{mk} = \partial_a \sigma^b_{mk} - \partial_b \sigma^a_{mk} + \sigma^a_{m\lambda}\sigma^b_{\lambda k} - \sigma^b_{m\lambda}\sigma^a_{\lambda k} =$$
$$\partial_a \Gamma^m_{bk} - \partial_b \Gamma^m_{ak} + \Gamma^m_{a\lambda}\Gamma^\lambda_{bk} - \Gamma^m_{b\lambda}\Gamma^\lambda_{ak} = R^m_{kab},$$

We introduce also a two-index matrix γ^{ab}, where

$$\gamma^{ab}_{mk} = g_{m\lambda}\sigma^{ab}_{\lambda k} = g_{m\lambda}(\partial_a \Gamma^\lambda_{bk} - \partial_b \Gamma^\lambda_{ak} + \Gamma^\lambda_{a\rho}\Gamma^\rho_{bk} - \Gamma^\lambda_{b\rho}\Gamma^\rho_{ak}) = g_{m\lambda} R^\lambda_{kab} = R_{mkab}$$

In the last expression R_{mkab} is the Riemann tensor with all lower indices.

Acting by the compatibility operator on the matrix e and taking into a consideration that e_0 is the integrable function, we obtain

$$[\partial\widetilde{\partial}]_{ab} e = \widetilde{e}^{-1}\gamma^{ab} = e\sigma^{ab}, \qquad [\partial\widetilde{\partial}]_{ab} e = ([\partial\widetilde{\partial}]_{ab}\Omega)e_0,$$

$$\Omega e_0 \sigma^{ab} = ([\partial\widetilde{\partial}]_{ab}\Omega)e_0.$$

The Riemann space-time becomes *flat* if the two-index matrix γ^{ab} (or the two-index matrix σ^{ab}) becomes zero matrix. From the last equations it follows

$$e\sigma^{ab}e^{-1} = ([\partial\widetilde{\partial}]_{ab}\Omega)\widetilde{\Omega}, \quad e_0\sigma^{ab}e_0^{-1} = \widetilde{\Omega}([\partial\widetilde{\partial}]_{ab}\Omega).$$

Therefore, the Riemannian spacetime becomes *flat* if and only if the orthogonal matrix Ω is *integrable*.

The integrability of the matrix function e_0 in the expression $\boxed{e = \Omega e_0}$ implies

$$[\partial\widetilde{\partial}]_{ab}\Omega = \Omega e_0 \sigma^{ab} e_0^{-1}.$$

From the last equation it follows that the orthogonal matrix Ω (or e) is integrable if and only if the two-index matrix σ^{ab} (or the two-index matrix γ^{ab}) is a zero matrix. The integrability of the orthogonal matrix Ω leads to the integrability of the frame matrix e and the vector Q.

3. THE ABSOLUTE RICCI, RIEMANN CURVATURE AND THE ABSOLUTE FORM OF THE BIANCHI IDENTITIES.

3.1. The *absolute* form of the Ricci matrix.

The *absolute* (invariant) matrix K, and the twice covariant matrix $\underline{K}$ are determined by the expressions:

$$\boxed{K = ([D\widetilde{D}]e)e^{-1} = ([D\widetilde{D}]\Omega)\widetilde{\Omega}, \qquad [D\widetilde{D}] = \widetilde{e}^{-1}[\partial\widetilde{\partial}]e^{-1}}.$$

$$\underline{K} = \widetilde{e}\, K\, e = \widetilde{e}[D\widetilde{D}]e = \widetilde{e}\,\widetilde{e}^{-1}\left[\underset{\downarrow}{\partial}\,\widetilde{\partial}\right]e^{-1}\,\underset{\uparrow}{e} = i(\alpha)\,\widetilde{i}\,(\beta)\,e^{-1}[\partial\widetilde{\partial}]_{\alpha\beta}\,e = i(\alpha)\,\widetilde{i}\,(\beta)\,\sigma^{\alpha\beta}$$

From the expressions for the elements of the two-index curvature matrix σ^{ab} and the Riemann tensor R^{m}_{kab},

$$\sigma^{ab}_{mk} = \partial_a \sigma^{b}_{mk} - \partial_b \sigma^{a}_{mk} + \sigma^{a}_{ms}\sigma^{b}_{sk} - \sigma^{b}_{ms}\sigma^{a}_{sk} = R^{m}_{kab} =$$

$$= \partial_a \Gamma^{m}_{bk} - \partial_b \Gamma^{m}_{ak} + \Gamma^{m}_{as}\Gamma^{s}_{bk} - \Gamma^{m}_{bs}\Gamma^{s}_{ak}$$

it follows,

$$\underline{K}_{mn} = \widetilde{i}\,(m)\,\underline{K}\,i(n) = \widetilde{i}\,(m)\,i(\alpha)\,\widetilde{i}\,(\beta)\,\sigma^{\alpha\beta}\,i(n) = \sigma^{m\beta}_{\beta n} = R^{\beta}_{nm\beta}$$

The last expression coincides with the definition of the Ricci matrix. Therefore we can consider the matrix *K* as the absolute form of the Ricci matrix.

Special notation. For the twice covariant Ricci matrix $\underline{K}$ special notation is used,

$$R \equiv \underline{K} = i(\alpha)\,\widetilde{i}\,(\beta)\,\sigma^{\alpha\beta} = i(\alpha)\,\widetilde{i}\,(\beta)\,g^{-1}\gamma^{\alpha\beta}$$

$$R_{mn} = \sigma^{m\beta}_{\beta n} = \partial_m \sigma^{\beta}_{\beta n} - \partial_\beta \sigma^{m}_{\beta n} + \sigma^{m}_{\beta\lambda}\sigma^{\beta}_{\lambda n} - \sigma^{\beta}_{\beta\lambda}\sigma^{m}_{\lambda n}) =$$

$$= \partial_m \Gamma^{\beta}_{\beta n} - \partial_\beta \Gamma^{\beta}_{mn} + \Gamma^{\beta}_{m\lambda}\Gamma^{\lambda}_{\phi n} - \Gamma^{\beta}_{\beta\lambda}\Gamma^{\lambda}_{mn}$$

3.2. The absolute Riemann curvature matrix.

The *absolute* two-index matrix $K^{mn} = \{K^{mn}_{ij}\}$ is determined by

$$K^{mn} \equiv ([D\widetilde{D}]_{mn}\, e)\, e^{-1} = (\widetilde{e}^{-1}\, i(\alpha)\,\widetilde{i}\,(\beta)\, e^{-1})_{mn}\, e\,\sigma^{\alpha\beta}\, e^{-1}.$$

$$K^{\mu\nu}_{\alpha\beta} = \widetilde{e}^{-1}_{\mu m}\; e^{-1}_{n\nu}\; e_{\alpha\rho}\,\sigma^{mn}_{\rho\gamma}\, e^{-1}_{\gamma\beta}$$

It is easy to obtain the following equation:

$$\widetilde{e}_{a\alpha}\;\widetilde{e}_{b\beta}\;\widetilde{e}_{c\mu}\;\widetilde{e}_{d\nu}\;K^{\mu\nu}_{\alpha\beta} = g_{a\rho}\sigma^{cd}_{\rho b} = g_{\underline{a}\underline{\rho}}\bar{R}^{\underline{\rho}}_{\underline{b}\underline{c}\underline{d}} = R_{\underline{a}\underline{b}\underline{c}\underline{d}},$$

from which it follows that the two-index matrix $K^{mn} = \{K_{ij}^{mn}\}$ is the absolute curvature matrix or the absolute matrix form of the Riemann-Christoffel tensor $R_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}$.

3.3. Relations between matrices and appropriate tensor expressions from the theory of relativity.

1. *The one-index Christoffel matrix of the first kind* $\gamma^c \equiv \tilde{e}\partial_c e$.

$$\gamma^c = \frac{1}{2}(\partial_c g + g\, i(c)\,\tilde{\partial} - \partial\,\tilde{i}\,(c)\, g),$$

$$\gamma_{ab}^c = \frac{1}{2}(\partial_c\, g_{ab} + \partial_b\, g_{ac} - \partial_a\, g_{bc}) = \Gamma_{a,cb}.$$

2. *The one-index Christoffel matrix of the second kind* $\sigma^m \equiv e^{-1}\partial_m e$.

$$\sigma^m = \frac{g^{-1}}{2}(\partial_m g + g\, i(m)\tilde{\partial} - \partial\,\tilde{i}\,(m)\, g),$$

$$\sigma_{\mu\beta}^m = \frac{g_{\mu\rho}^{-1}}{2}\, g_{\mu\rho}^{-1}\,(\partial_\beta\, g_{m\rho} + \partial_m\, g_{\rho\beta} - \partial_\rho\, g_{m\beta}) = \Gamma_{m\beta}^{\mu}$$

3. *Ricci matrix* $R \equiv \underline{K} = i(\alpha)\,\tilde{i}\,(\beta)\,\sigma^{\alpha\beta} = i(\alpha)\,\tilde{i}\,(\beta)\, g^{-1}\gamma^{\alpha\beta}$,

$$R_{mn} = \sigma_{\beta n}^{m\beta} = R_{nm\beta}^{\beta},$$

$$\sigma_{\beta n}^{m\beta} = \partial_m \sigma_{\beta n}^{\beta} - \partial_\beta \sigma_{\beta n}^{m} + \sigma_{\beta\lambda}^{m}\sigma_{\lambda n}^{\beta} - \sigma_{\beta\lambda}^{\beta}\sigma_{\lambda n}^{m} = R_{n\,m\beta}^{\beta} =$$

$$= \partial_m \Gamma_{\beta n}^{\beta} - \partial_\beta \Gamma_{m\,n}^{\beta} + \Gamma_{m\lambda}^{\beta}\,\Gamma_{\beta n}^{\lambda} - \Gamma_{\beta\lambda}^{\beta}\,\Gamma_{mn}^{\lambda}$$

4. *The absolute Riemann curvature matrix.*

$$K^{mn} \equiv ([D\,\tilde{D}]_{mn}\, e)\, e^{-1} = (\tilde{e}^{-1}\, i(\alpha)\,\tilde{i}\,(\beta)\, e^{-1})_{mn}\, e\,\sigma^{\alpha\beta}\, e^{-1}$$

$$\tilde{e}_{a\alpha}\,\tilde{e}_{b\beta}\,\tilde{e}_{c\mu}\,\tilde{e}_{d\nu}\, K_{\alpha\beta}^{\mu\nu} = g_{a\rho}\sigma_{\rho b}^{cd} = g_{\underline{a}\,\underline{\rho}}\bar{R}_{\underline{b}\,\underline{c}\,\underline{d}}^{\rho} = R_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}$$

3.4. Properties of the absolute Riemann curvature and Ricci matrices.

We have obtained the following properties of the absolute Riemann curvature and Ricci matrices:

$$1.\ K^{mn} + \tilde{K}^{mn} = 0$$

$$2.\ K^{mn} i(k) + K^{nk} i(m) + K^{km} i(n) = 0$$

$$3.\ K^{mn}_{rk} = K^{rk}_{mn}.$$

$$4.\ K^{\lambda n}_{\beta\lambda} = K^{n\lambda}_{\lambda\beta} = K_{n\beta} = K_{\beta n}.$$

$$5.\ K = \widetilde{K}.$$

$$6.\ \underset{\uparrow}{K}\,\underset{\downarrow}{D} = D_\lambda K\, i(\lambda) = \frac{1}{2} D\langle K\rangle.$$

$$7.\ D_k K^{mn} + D_m K^{nk} + D_n K^{km} = 0.$$

4. THE FIELD MATRIX IN N–DIMENSIONAL METRIC SPACE. FIELD EQUATIONS

4.1 The absolute velocity vector and the absolute field matrix.

The *absolute* (non-integrable) and coordinate (integrable) velocity vectors.

Using the notations: *U* for the *absolute* velocity-vector and *u(x)* for the *coordinate* velocity-vector, we can write the following definitions and expressions

$$\boxed{U = u(Q) \equiv \frac{dQ}{d\tau} = e\,u(x), \quad u(x) \equiv \frac{dx}{d\tau}, \quad d\tau^2 = d\widetilde{x}\, g\, dx = d\widetilde{Q}\, dQ}$$

From the expressions for the infinitesimal invariant interval $d\tau$ it follows,

$$\widetilde{U}U = \widetilde{u}(x)\, g\, u(x) = 1.$$

4.2 The absolute non-integrable but differentiable field matrix.

The exact differential *dU* of the *absolute* velocity vector *U* is written as

$$dU = Ud = (\underset{\uparrow}{U}\,\underset{\downarrow}{\widetilde{D}})dQ.$$

Then it is possible to write

$$\boxed{\frac{dU}{d\tau} = (\underset{\uparrow}{U}\,\underset{\downarrow}{\widetilde{D}})\frac{dQ}{d\tau} = (\underset{\uparrow}{U}\,\underset{\downarrow}{\widetilde{D}})U = PU, \quad dU = P\,dQ, \quad P \equiv \underset{\uparrow}{U}\,\underset{\downarrow}{\widetilde{D}},}$$

where the *absolute* matrix *P* as the *absolute* field matrix.

4.3 The compatibility conditions for the vector *U*.

We know that in order the differential $dQ = e\,dx$ be exact differential, it is necessary and sufficient that the compatibility conditions are fulfilled for the vector *Q*:

$$\partial_k(x)ei(j)-\partial_j(x)ei(k)=0,\quad k,j=1,...,n$$

Therefore in order that the differential $dU=P\,dQ$ be exact differential, it is necessary and sufficient that the compatibility conditions are fulfilled for the vector *U*:

$$\left[D\tilde{D}\right]_{kj}U=0,$$

$$P=U\tilde{D},\quad \partial_k(Q)Pi(j)-\partial_j(Q)Pi(k)=D_k\,Pi(j)-D_j\,Pi(k)=0,\quad k,j=1,...,n$$

4.4 Two types of absolute mathematical field matrices: the absolute symmetric and the absolute antisymmetric mathematical field matrices.

The *absolute* matrix *P* can be written as the sum of two matrices

$$\underset{\uparrow}{U}\,\underset{\downarrow}{\tilde{D}}=P=P_a+P_s,\qquad P_a\equiv\frac{1}{2}(\underset{\uparrow}{U}\,\underset{\downarrow}{\tilde{D}}-D\tilde{U})=-\tilde{P}_a,$$

$$P_s\equiv\frac{1}{2}(\underset{\uparrow}{U}\,\underset{\downarrow}{\tilde{D}}+D\tilde{U})=\tilde{P}_s$$

where P_s and P_a are the absolute symmetric and antisymmetric field matrices.

4.5 The *absolute* equation of motion.

From the equations

$$\tilde{U}U=1,\quad \tilde{U}\,P_a\,U=0,$$

we obtain

$$\frac{d(\tilde{U}U)}{d\tau}=\tilde{U}(\tilde{P}_s+\tilde{P}_a)U+\tilde{U}(P_s+P_a)U=2\tilde{U}P_sU=0,$$

That means the symmetric *absolute* field matrix must satisfy the condition $\boxed{P_sU=0}$.

Therefore the *absolute* equation of motion is written in the form

$$\boxed{\frac{dU}{d\tau}=(\underset{\uparrow}{U}\,\underset{\downarrow}{\tilde{D}})U=(P_a+P_s)U=P_aU}.$$

4.6 The coordinate equation of motion.

Substituting into the *absolute* equation of motion the following expressions,

$$\frac{dU}{d\tau}=\frac{de}{d\tau}u+e\frac{du}{d\tau},\qquad \frac{de}{d\tau}=e\sigma^{\mu}u_{\mu},\qquad P_aU=\tilde{e}^{-1}\underline{P_a}u,$$

we obtain

$$\frac{du}{d\tau}=-\sigma^{\mu}u_{\mu}u+g^{-1}\underline{P_a}u.$$

We call the last equation the *coordinate* equation of motion, where

$$\sigma^{\mu}=i(\alpha)\tilde{i}(\beta)\Gamma^{\alpha}_{\mu\beta},\quad \sigma^{\mu}_{\beta\alpha}=\Gamma^{\beta}_{\mu\alpha}.$$

The scalar *coordinate* equation of motion

$$\frac{du_{\beta}(x)}{d\tau}=g^{-1}_{\beta\lambda}\underline{P}_{a\lambda\mu}\,u_{\mu}(x)-\sigma^{\mu}_{\beta\alpha}u_{\mu}(x)u_{\alpha}(x)=g^{-1}_{\beta\lambda}\underline{P}_{a\lambda\mu}\,u_{\mu}(x)-\Gamma^{\beta}_{\mu\alpha}u_{\mu}(x)u_{\alpha}(x)$$

for $P_a = 0$ becomes

$$\boxed{\frac{du_{\beta}(x)}{d\tau}=-\sigma^{\mu}_{\beta\alpha}u_{\mu}(x)u_{\alpha}(x)=-\Gamma^{\beta}_{\mu\alpha}u_{\mu}(x)u_{\alpha}(x)},$$

which is the differential equation of geodesic, well known from the tensor classical theory of general relativity.

4.7 The 4-dimensional space: the coordinate equation of motion of a charged particle in the presence of both gravitational and electromagnetic fields.

The coordinate equation of motion of a charged particle in the presence of both gravitational and electromagnetic fields is written in the form [1]:

$$\frac{du_{\beta}(x)}{d\tau}=\frac{e}{mc^2}F^{\beta}{}_{\mu}\,u_{\mu}(x)-\Gamma^{\beta}_{\mu\alpha}u_{\mu}(x)u_{\alpha}(x),$$

where $F^{\beta}{}_{\mu}$ is the tensor of an electromagnetic field. If we compare the last equation with the scalar *coordinate* equation of motion

$$\frac{du_{\beta}(x)}{d\tau}=g^{-1}_{\beta\lambda}\underline{P}_{a\lambda\mu}\,u_{\mu}(x)-\sigma^{\mu}_{\beta\alpha}u_{\mu}(x)u_{\alpha}(x)=g^{-1}_{\beta\lambda}\underline{P}_{a\lambda\mu}\,u_{\mu}(x)-\Gamma^{\beta}_{\mu\alpha}u_{\mu}(x)u_{\alpha}(x)$$

,

we see that the antisymmetric matrix $\vec{P}_a$, $\boxed{\vec{P}_a=g^{-1}\underline{P}_a}$, is the matrix of the electromagnetic field.

If the antisymmetric field matrix P_a is zero and there is only the symmetric field matrix P_s, the equation of motion becomes the following one:

$$\frac{du_\beta(x)}{d\tau} = -\sigma^{\mu}_{\beta\alpha} u_\mu(x) u_\alpha(x) = -\Gamma^{\beta}_{\mu\alpha} u_\mu(x) u_\alpha(x),$$

which in the four-dimensional space is the equation of motion in a gravitational field. Therefore, the symmetric field matrix P_s in the four-dimensional space is the matrix of a gravitational field and the field matrix P can be considered as an electromagnetic-gravitational field matrix.

5. THE FIELD MATRIX IDENTITIES AND DERIVATION OF THE EQUATIONS OF THE FIELD.

5.1 The first field matrix identity.

The first field matrix identity connects the absolute Ricci matrix K with the *absolute field matrix* P. The coordinate vector-velocity $u(x)$ is an integrable function. Therefore

$$[D\,\widetilde{D}]\,u(x) = 0.$$

Using the last equation, the expression for $\boxed{[D\,\widetilde{D}]U}$ can be written in two forms. The first form is

$$\boxed{[D\,\widetilde{D}]\,U = ([D\,\widetilde{D}]\,e)u(x) = ([D\,\widetilde{D}]\,e)e^{-1}\,U = K\,U},$$

and the second form is

$$\boxed{[D\,\widetilde{D}]U = D\langle U\,\widetilde{D}\rangle - (D\widetilde{U})\,D = D\langle P\rangle - (\widetilde{P})\,D}.$$

$$P_{\lambda\beta} = U_\lambda\, D_\beta, \quad \widetilde{P}_{\lambda\beta} = D_\lambda\, U_\beta$$

$$\boxed{[D\,\widetilde{D}]_{\lambda\beta}\,U_\beta = D_\lambda\,(D_\beta\,U_\beta) - D_\beta\,(D_\lambda\,U_\beta) = D_\lambda\,(P_{\beta\beta}) - (\widetilde{P}_{\lambda\beta})D_\beta}$$

Therefore we obtain the first *absolute* field matrix identity,

$$\boxed{KU = D\langle P\rangle - (\widetilde{P})D = D\langle P_s\rangle + (P_a - P_s)D},$$

which has the following coordinate form

$$\underline{K}\,u(x) = \partial(x)\left\langle g^{-1}\,\underline{P_s}\right\rangle + (\widetilde{\sigma}^{\beta}\,(\underline{P_s} - \underline{P_a}) - \partial_{\beta}(x)(\underline{P_s} - \underline{P_a}) + (\underline{P_s} - \underline{P_a})\,\sigma^{\beta})\,g^{-1}\,i(\beta)$$

$$\underline{K}\,u(x) = \partial(x)\left\langle \overline{P_s}\,g\right\rangle - g(\sigma^{\beta}\,(\overline{P_s} - \overline{P_a}) + \partial_{\beta}(x)\,(\overline{P_s} - \overline{P_a}) + (\overline{P_s} - \overline{P_a})\,\widetilde{\sigma}^{\beta})\,i(\beta)$$

where

$$\langle P\rangle = \left\langle g^{-1}\,\underline{P_s}\right\rangle = \left\langle \overline{P_s}\,g\right\rangle,$$

$$\underline{P_s} - \underline{P_a} = i(\beta)(\partial_{\beta}\,\widetilde{u}(x) + \widetilde{u}(x)\widetilde{\sigma}^{\beta})\,g,$$

$$\overline{P_s} - \overline{P_a} = g^{-1}i(\beta)(\partial_{\beta}\,\widetilde{u}(x) + \widetilde{u}(x)\widetilde{\sigma}^{\beta})\,g$$

5.2 The first group of the equations of a field.

Now we will show that the absolute expression KU can be written in the form,

$$KU = \mu(K)U,$$

where $\mu(K)$ is an eigenvalue of the *absolute* Ricci matrix. It can be done in two different cases.

In the first case all *n* eigenvalues $\mu(K)$ are equal, satisfying the following equations

$$\mu_1(K) = \mu_2(K) = ... = \mu_n(K) = \mu,$$

and the metric matrix *g* corresponds to these equations. In the second case there is such absolute velocity vector *U*, which is equal to the eigenvector $U(\mu_j(K))$,

$$U = U(\mu_j(K)).$$

In the first case the appropriate absolute and coordinate Ricci matrices satisfy the matrix equations,

$$K = \widetilde{e}^{-1}\,\underline{K}\,e^{-1} = \mu I, \quad R = \underline{K} = \mu g, \quad g^{-1}R = \mu I,$$

where the twice covariant Ricci matrix $\underline{K} = R$ is connected with the absolute Ricci matrix K,

$$\underline{K} = \tilde{e} K e .$$

At this case we can write

$$KU = \mu U, \quad g^{-1} R\, u(x) = \mu\, u(x),$$

where $\mu = \mu_1(K) = \mu_2(K) = ... = \mu_n(K)$, or

$\mu = \mu_1(g^{-1} R) = \mu_2(g^{-1} R) = ... = \mu_n(g^{-1} R)$

In the second case

$$KU = \mu U, \quad g^{-1} R\, u(x) = \mu\, u(x),$$

where $\mu = \mu_j(K)$, or $\mu = \mu_j(g^{-1} R)$ and $U = U(\mu_j(K))$.

In the both cases we call the field equations as the eigenvalues field equations

$$KU = \mu U, \quad g^{-1} R\, u(x) = \mu\, u(x)$$

In the first case the eigenvalues field equation is written also in the form:

$$R = \underline{K} = \mu g, \quad g^{-1} R = \mu I .$$

The first group of the *absolute* equations of the field, written in the form

$$\boxed{D\langle P\rangle - (\tilde{P}) D = \mu U},$$

reminds the Maxwell equation for the electromagnetic field. For the antisymmetric absolute field in four-dimensional space it becomes the Maxwell equation of an antisymmetric field matrix P_a,

$$\boxed{P_a D = \mu_a U}.$$

In the 4 – dimensional space the antisymmetric field matrix becomes the electromagnetic field matrix and the last equation will be Maxwell equation for the electromagnetic field.

A. Einstein wrote [2]: "The only clue which can be drawn from experience is the vague perception that something like Maxwell's electromagnetic field has to be contained within the total field".

The coordinate form of the absolute equation of the field

$$\boxed{D\langle P\rangle - (\tilde{P}) D = \mu U}$$

is written for the general field matrix *P* as

$$R\,u(x) = \partial(x)\left\langle g^{-1}\,\underline{P_s}\right\rangle + (\tilde{\sigma}^{\alpha}\,(\underline{P_s} - \underline{P_a}) - \partial_{\alpha}(x)(\underline{P_s} - \underline{P_a}) +$$
$$(\underline{P_s} - \underline{P_a})\,\sigma^{\alpha})\,g^{-1}\,i(\alpha) = \mu\,g\,u(x)$$

for an antisymmetrical field matrix as

$$(-\tilde{\sigma}^{\alpha}(g_a)\underline{P_a} + \partial_{\alpha}(x)\underline{P_a} - \underline{P_a}\,\sigma^{\alpha}(g_a)\,g_a^{-1}\,i(\alpha) = \mu_a\,g_a\,u(x)$$

and for a symmetrical field matrix as

$$\partial(x)\left\langle g_s^{-1}\,\underline{P_s}\right\rangle + (\tilde{\sigma}^{\alpha}(g_s)\underline{P_s} - \partial_{\alpha}(x)\underline{P_s} + \underline{P_s}\,\sigma^{\alpha}(g_s))\,g_s\,i(\alpha) = \mu_s\,g_s\,u(x).$$

5.3 The second field matrix identity.

The second field matrix identity connects the Riemann curvature matrix K^{mn} with the *absolute field matrix P*. The expression for $[D\tilde{D}]_{mn}U$ can be written in two forms,

$$\boxed{[D\tilde{D}]_{mn}\,U = K^{mn}\,U},$$

and

$$\boxed{[D\tilde{D}]_{mn}\,U = D_m\,Pi(n) - D_n\,Pi(m)}.$$

From these two expressions it follows the *second absolute field matrix identity,*

$$\boxed{K^{mn}\,U = D_m\,Pi(n) - D_n\,Pi(m)}.$$

where

$$\boxed{P = \underset{\uparrow}{U}\,\underset{\downarrow}{\tilde{D}}}.$$

The *second absolute field matrix identity* can be written also in the form:

$$K^{mn}\,U_k = [D\tilde{D}]_{mn}\,U_k = D_m\,D_n\,U_k - D_n\,D_m\,U_k,$$

where

$$K^{mn} = (\tilde{e}^{-1}\,i(\alpha)\,\tilde{i}\,(\beta)\,e^{-1})_{mn}\,e\,\sigma^{\alpha\beta}\,e^{-1}.$$

5.4 The second group of the equations of the field.

Using the *absolute* identity

$$\tilde{i}(k)K^{mn} + \tilde{i}(m)K^{nk} + \tilde{i}(n)K^{km} = 0,$$

and the second *absolute* field matrix identity

$$\boxed{K^{mn}U = D_m Pi(n) - D_n Pi(m)}$$

we obtain the second group of the *absolute* field equations for the antisymmetric field matrix,

$$\boxed{D_m P_{akn} + D_n P_{amk} + D_k P_{anm} = 0},$$

because

$$\tilde{i}(k)K^{mn}U + \tilde{i}(m)K^{nk}U + \tilde{i}(n)K^{km}U =$$

$$= D_m P_{kn} - D_n P_{km} + D_n P_{mk} - D_k P_{mn} + D_k P_{nm} - D_m P_{nk} = .$$

$$= 2(D_m \tilde{i}(k)P_a i(n) + D_n \tilde{i}(m)P_a i(k) + D_k \tilde{i}(n)P_a i(m)) = 0$$

The second group of the *absolute* field equations for the symmetric field matrix is

$$\boxed{P_s U = 0}.$$

5.5 The partial differential equation for the metric matrix *g*.

From the previous results we know the following expressions

$$R = i(\alpha)\tilde{i}(\beta)\sigma^{\alpha\beta},$$

$$\sigma^{ab} \equiv \partial_a \sigma^b - \partial_b \sigma^a + \sigma^a \sigma^b - \sigma^b \sigma^a$$

$$\sigma^m \equiv e^{-1}\partial_m e = g^{-1}\gamma^m$$

$$\gamma^m = \frac{1}{2}(\partial_m g + g\, i(m)\tilde{\partial} - \partial \tilde{i}(m)\, g)$$

Using the equation of the mathematical field

$$R = \mu\, g,$$

we obtain the partial differential equation for the metric matrix g :

$$\mu\, g = i(\alpha)\tilde{i}(\beta)\sigma^{\alpha\beta},$$

$$R_{mn} = \mu\, g_{mn} = \sigma^{m\beta}_{\beta n} = \partial_m \Gamma^{\beta}_{\beta n} - \partial_\beta \Gamma^{\beta}_{mn} + \Gamma^{\beta}_{m\lambda}\Gamma^{\lambda}_{\phi n} - \Gamma^{\beta}_{\beta\lambda}\Gamma^{\lambda}_{mn}$$

where in the right part there are different partial derivatives of the metric matrix g,

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}\, g^{-1}_{\mu j}\,(\partial_{\beta}\, g_{\alpha j} + \partial_{\alpha}\, g_{j\beta c} - \partial_{j}\, g_{\alpha\beta})\,.$$

5.6 The invariant absolute divergence of the vector $\mu\, U$.

The invariant absolute divergence of the vector $\mu\, U$ is written in the form,

$$\widetilde{D}(\mu U) = \underset{\downarrow}{\widetilde{\partial}} e^{-1}\,(e(\mu\, \underset{\uparrow}{u}(x))) = \widetilde{i}\,(\lambda)(\sigma^{\lambda}\,\mu\, u(x) + \partial_{\lambda}(\mu\, u(x))$$

$$\widetilde{D}(\mu\, U) = \frac{d\mu}{d\tau} + \mu\langle P\rangle = \frac{d\mu}{d\tau} + \mu\langle \overline{P}\, g\rangle\,,$$

5.7 The invariant absolute divergence of the vector ρU for the antisymmetric field matrix.

For the field matrix we have

$$\mu U = K U = D\langle P\rangle - (\widetilde{P})D\,.$$

For the antisymmetric field matrix we obtain

$$\widetilde{D}(\mu_a\, U) = \frac{d\,\mu}{d\tau} = \widetilde{D}(P_a D) = 0\,.$$

5.8 The invariant absolute and coordinate divergence of the vector μU for the symmetric field matrix.

For the symmetric field matrix it follows from the previous equations,

$$\widetilde{D}(\mu_s\, U) = \widetilde{D}(D\langle P_s\rangle - P_s D)\,,$$

$$\widetilde{D}(\mu_s\, U) = (\widetilde{D}\mu_s)U + \mu_s\langle P_s\rangle = \frac{d\mu_s}{d\tau} + \mu_s\langle P_s\rangle\,.$$

For the electromagnetic field the absolute field matrix P_a is an antisymmetric matrix. Therefore the equation of continuity,

$$\widetilde{D}(\mu_a\, U) = \frac{d\,\mu_a}{d\tau} = \widetilde{D}(P_a D) = 0$$

is an expression of the law of charge conservation

$$\boxed{\tilde{D}(\mu_a U) = \tilde{D}(P_a D)) = \frac{d\mu_a}{d\tau} = 0}.$$

PART 2. 4-DIMENSIONAL METRIC SPACE

1. THE FIELD MATRIX IN THE 4-DIMENSIONAL SPACE AS THE ELECTROMAGNETIC-GRAVITATIONAL FIELD MATRIX

The coordinate equation of motion of a charged particle in the presence of both gravitational and electromagnetic fields is written in the form [1]:

$$\frac{du_\beta(x)}{d\tau} = \frac{e}{mc^2} F^\beta{}_\mu \, u_\mu(x) - \Gamma^\beta_{\mu\alpha} u_\mu(x) u_\alpha(x),$$

where $F^\beta{}_\mu$ is the tensor of an electromagnetic field. If we compare the last equation with the scalar *coordinate* equation of motion

$$\frac{du_\beta(x)}{d\tau} = g^{-1}_{\beta\lambda} \underline{P}_{a\lambda\mu} \, u_\mu(x) - \sigma^\mu_{\beta\alpha} u_\mu(x) u_\alpha(x) = g^{-1}_{\beta\lambda} \underline{P}_{a\lambda\mu} \, u_\mu(x) - \Gamma^\beta_{\mu\alpha} u_\mu(x) u_\alpha(x)$$

we see that the antisymmetric matrix $\vec{P}_a$, where

$$\boxed{\vec{P}_a = g^{-1} \underline{P}_a}$$

is the matrix of an electromagnetic field.

If the antisymmetric field matrix P_a is zero and there is only the symmetric field matrix P_s, the equation of motion becomes the following one:

$$\frac{du_\beta(x)}{d\tau} = -\sigma^\mu_{\beta\alpha} u_\mu(x) u_\alpha(x) = -\Gamma^\beta_{\mu\alpha} u_\mu(x) u_\alpha(x),$$

which in the four-dimensional space is the equation of motion in the gravitational field. Therefore, the symmetric field matrix P_s in the four-dimensional space is the matrix of the gravitational field and the field matrix P can be considered as the electromagnetic-gravitational field matrix.

2. THE MASS FUNCTION m_f AND THE MASS DENSITY FUNCTION ρ_f.

Let us apply our matrix theory for finding the spherically symmetric solution of the metric matrix equation in the four-dimensional metric space. We will use the following notations:

$$m_g = m[g]\, g,$$

where m_g is the mass in grams, $m[g]$ is the number of grams.

For example,

$$m_g = 5.5\, g,\ m[g] = 5.5;$$

Newton's gravitational constant $G = 6.6743 \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}$;

$$G = G[cm^3\ g^{-1}\ s^{-2}]\, cm^3\ g^{-1}\ s^{-2};$$

$$G[cm^3\ g^{-1}\ s^{-2}] = 6.6743 * 10^{-8},$$

$$c = 2.99792458 \times 10^{10}\ cm\ s^{-1};$$

$$c[cm\ s^{-1}] = 2.99792458 \times 10^{10};$$

$$\frac{G}{c^2} = \frac{G}{c^2}[cm\ g^{-1}]\ cm\, g^{-1};$$

$$\frac{G}{c^2}[cm\, g^{-1}] = 0.7426 * 10^{-28};$$

$$m_f = \alpha_m \frac{Gm}{c^2}[cm]cm.$$

$$\rho_f = \frac{G}{c^2}\rho = \frac{G}{c^2}[cm\, g^{-1}]\ cm\ g^{-1}\, \rho[g\ cm^{-3}]\ g\ cm^{-3} = (\frac{G}{c^2}\rho)[cm^{-2}]cm^{-2} =$$

$$= \rho_f\ [cm^{-2}]cm^{-2}$$

Here m_f is the mass function for the mass m, ρ_f is the mass density function for the mass density ρ, α_m is the dimensionless constant for the mass m. For the concentrated mass M of Sun $\alpha_M = 2$.

3. THE GENERAL SPHERICALLY SYMMETRIC SOLUTION OF THE METRIC MATRIX EQUATION.

Let us apply our matrix theory for finding the spherically symmetric solution of the metric matrix equation for the symmetric field matrix in the four-dimensional metric space.

3.1 THE "RECTILINEAR" AND "SPHERICAL" COORDINATE SYSTEMS.

We use the "rectilinear" coordinate system with the four-dimensional vector *y* and the corresponding "spherical" coordinate system with the four-dimensional vector *x*, where

$$y_1 = x_3 \cos x_1, \quad y_2 = x_3 \sin x_1 \sin x_2,$$
$$y_3 = x_3 \sin x_1 \cos x_2, \quad y_4 = x_4 = ct$$

Here *c* is the speed of light, *t* is time,

$$x_3 = \sqrt{y_1^2 + y_2^2 + y_3^2} = r, \quad x_1 = \vartheta, \quad x_2 = \varphi.$$

3.2 THE GENERALIZED SCHWARZSCHILD SOLUTION OF THE METRIC MATRIX EQUATION

The spatial interval and the metric matrix with spherical symmetry, which used by Schwarzschild, are written in the forms:

$$d\tau^2 = -g_1(x_3)x_3^2\,dx_1^2 - g_1(x_3)x_3^2 \sin^2 x_1\,dx_2^2 - g_{33}(x_3)dx_3^2 + g_{44}(x_3)dx_4^2,$$

$$g = \{\{-g_1(x_3)x_3^2, 0, 0, 0\}, \{0, -g_1(x_3)x_3^2 \sin^2 x_1, 0, 0\}, \{0, 0, -$$
$$-g_{33}(x_3), 0\}, \{0, 0, 0, g_{44}(x_3)\}\}$$

The unknown functions $g_1(x_3)$, $g_{33}(x_3)$ and $g_{44}(x_3)$ are to be determined by solving our differential equations $\mu_\alpha - \mu_\beta = 0$, where μ_α and μ_β are the eigenvalues of the matrix $g^{-1}R$, *g* is the metric matrix and *R* is the Ricci matrix. Determinant of this metric matrix *g* is the same as the determinant of the metric matrix of the flat space time:

$$Det(g) = -x_3^4 \sin^2 x_1 = -x_3^4 \sin^2 x_1\, g_{33}(x_3)\, g_{44}(x_3)\, g_1^2(x_3).$$

From the last equation we obtain

$$g_{44}(x_3) = \frac{1}{g_1^2(x_3)\, g_{33}(x_3)}.$$

Sometimes we will use short notations, omitting argument.
For our metric matrix *g* we find

$$\mu_1 = \mu_2 = \alpha(x_3^2\, g_{33}(g_1')^2 - 2g_1^2(g_{33} - x_3\, g_{33}') + g_1(2g_{33}^2 + x_3^2\, g_1'\, g_{33}' -$$
$$-x_3\, g_{33}(2g_1' + x_3\, g_1'')))$$

$$\alpha = \frac{1}{2x_3^2\, g_1^2\, g_{33}^2}$$

$$\mu_3 = \frac{\alpha}{g_{33}}(-3x_3\, g_{33}^2 (g_1^{'})^2 - 2g_1 g_{33} g_1^{'}(2g_{33} + x_3 g_{33}^{'}) + g_1^2(2g_{33}^{'}(g_{33} - x_3 g_{33}^{'} + x_3 g_{33} g_{33}^{''}))$$

$$\mu_4 = \frac{\alpha}{g_{33}}(-2x_3\, g_{33}^2 (g_1^{'})^2 + 2g_1 g_{33}(g_1^{'}(2g_{33} - x_3 g_{33}^{'}) + x_3\, g_{33} g_1^{''}) + g_1^2(2g_{33}^{'}(g_{33} -$$

$$- x_3 g_{33}^{'} + x_3 g_{33} g_{33}^{''}))$$

$$\mu_4 - \mu_3 = \frac{8g_1(x_3)g_1^{'}(x_3) + x_3(g_1^{'}(x_3))^2 + 2x_3 g_1(x_3) g_1^{''}(x_3)}{2x_3\, g_1^2(x_3)\, g_{33}(x_3)}$$

The solution of the differential equation

$$\mu_4 - \mu_3 = 0$$

is the following one:

$$g_1 = g_1(x_3) = w^{2/3}$$

where

$$w = w(x_3) = 1 - \frac{c_5^3}{x_3^3}.$$

Here c_5 is constant. The interpretation of this constant is m_f, the mass function for the moving mass m.

$$m_f = \alpha_m \frac{Gm}{c^2}[cm]cm.$$

Now we can write the expression for $\mu_1 - \mu_3$.

$$\mu_1 - \mu_3 = \frac{1}{x_3^2\, g_1} + \frac{-1 + 8\frac{c_5^3}{x_3^3} + 2\frac{c_5^6}{x_3^6}}{x_3^2(1 - \frac{c_5^3}{x_3^3})^2\, g_{33}} + \frac{(g_{33}^{'})^2}{g_{33}^3} + \frac{\frac{3c_5^3\, g_{33}^{'}}{x_3^4(1 - \frac{c_5^3}{x_3^3})} - \frac{g_{33}^{''}}{2}}{g_{33}^2}$$

We obtain the solution of the differential equation $\mu_1 - \mu_3 = 0$ in the form:

$$g_{33} = \frac{1}{w(w^{1/3} - \frac{c_6}{x_3} + c_7 x_3^2)}$$

Now we can find g_{44}.

$$g_{44} = \frac{1}{g_1^2 g_{33}} = 1 + \frac{-\frac{c_6}{x_3} + c_7 x_3^2}{w^{1/3}}$$

For the obtained metric matrix we can determine different eigenvalues μ through the mass density function ρ_f, mass density function for the mass density ρ,

$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \rho_f = -3c_7$$

Therefore, the interpretation of constant c_7 is the mass density function $c_7 = -\frac{\rho_f}{3}$.

The interpretation of constant c_6 is M_f, the mass function for the concentrated mass M,

$$M_f = \alpha_m \frac{GM}{c^2}[cm]cm, \quad \alpha_M = 2..$$

This function is called also as Schwarzschild radius 2 r_M, where

$$r_M = M\frac{G}{c^2}.$$

Sometimes the concentrated mass M is called the central mass or the mass of the gravitating object.

For our metric matrix,

$g = \{\{-g_1(x_3)x_3^2, 0, 0, 0\}, \{0, -g_1(x_3)x_3^2 \sin^2 x_1, 0, 0\}, \{0, 0, -$

$-g_{33}(x_3), 0\}, \{0, 0, 0, g_{44}(x_3)\}\}$

using the short notice,

$$w = 1 - \frac{c_5^3}{x_3^3}$$

we will write the coefficients, obtained for our generalized Schwarzschild solution,

$$g_1 = g_1(x_3) = w^{2/3}$$

$$g_{33} = \frac{1}{w(w^{1/3} - \frac{2r_M}{x_3} - \frac{\rho_f}{3}x_3^2)}$$

$$g_{44} = 1 - \frac{\frac{2r_M}{x_3} + \frac{\rho_f}{3}x_3^2}{w^{1/3}}$$

For $c_5 = 0, \quad \rho_f = 0$ we obtain the Schwarzschild solution of the metric,

$$w = 1, \quad g_1 = 1, \quad g_{33} = \frac{1}{1 - \frac{2r_M}{x_3}}, \quad g_{44} = 1 - \frac{2r_M}{x_3}.$$

3.3 RICCI MATRIX FOR THE GENERALIZED SCHWARZSCHILD SOLUTION.

For the obtained generalized Schwarzschild solution Ricci matrix is determined by the expression:

$$R = -3c_7\, g = \rho_f\, g\,.$$

Schwarzschild has found his solution, solving the equation $R = 0$. It is easy to see, that $R = 0$, if $\rho_f = 0$. Therefore $R = 0$ for three different metric matrices:

1. The Schwarzschild metric: $w = 1, \quad g_1 = 1, \quad g_{33} = \frac{1}{1 - \frac{2r_M}{x_3}}, \quad g_{44} = 1 - \frac{2r_M}{x_3}.$

2. The metric:

$$w = 1 - \frac{c_5^3}{x_3^3}, \quad g_1 = g_1(x_3) = w^{2/3}, \quad g_{33} = \frac{1}{w(w^{1/3} - \frac{2r_M}{x_3})}, \quad g_{44} = 1 - \frac{2r_M}{x_3\, w^{1/3}}.$$

3. The metric:

$$w = 1 - \frac{c_5^3}{x_3^3}, \quad g_1 = g_1(x_3) = w^{2/3}, \quad g_{33} = \frac{1}{w^{4/3}}, \quad g_{44} = 1.$$

3.4 CURVATURE MATRICES FOR THE GENERALIZED SCHWARZSCHILD SOLUTION.

Let us obtain the two index curvature matrices $\sigma^{1,2}$, $\sigma^{1,3}$, $\sigma^{1,4}$, $\sigma^{2,3}$, $\sigma^{2,4}$, $\sigma^{3,4}$ for the generalized Schwarzschild metric. We find then:

$$\sigma^{12} = \{\{0,(1-g_{44})\sin^2 x_1,0,0\},\{g_{44}-1,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}$$

$$g_{44}-1 = \frac{-\frac{c_6}{x_3}+c_7 x_3^2}{w^{1/3}}$$

$$\sigma^{13} = \{\{0,0,\frac{g_5}{2x_3^2 w^{7/3} g_{44}},0\},\{0,0,0,0\},\{\frac{g_5}{2w^{1/3}},0,0,0\},\{0,0,0,0\}\}$$

$$g_5 = \frac{c_6}{x_3}+(2-3\frac{c_5^3}{x_3^3})x_3^2 c_7$$

$$\sigma^{14} = \{\{0,0,0,\frac{g_5 g_{44}}{2x_3^2 w}\},\{0,0,0,0\},\{0,0,0,0\},\{\frac{g_5}{2w},0,0,0\}\}$$

$$\sigma^{23} = \{\{0,0,0,0\},\{0,0,-\frac{g_5}{2x_3^2 w^{7/3} g_{44}},0\},\{0,\frac{g_5\sin^2 x_1}{2w^{1/3}},0,0\},\{0,0,0,0\}\}$$

$$\sigma^{24} = \{\{0,0,0,0\},\{0,0,0,\frac{g_5 g_{44}}{2x_3^2 w}\},\{0,0,0,0\},\{0,\frac{g_5\sin^2 x_1}{2w^{1/3}},0,0\}\}$$

$$\sigma^{34} = \{\{0,0,0,0,0\},\{0,0,0,0\},\{0,0,0,-\frac{(1-g_{44})g_{44}}{x_3^2 w^{2/3}}\},\{0,0,\frac{g_{44}-1}{x_3^2 w^2 g_{44}},0\}\}$$

We see that all curvature matrices are proportional to the concentrated mass and mass density. That means the curvature of the space is produced only by the concentrated mass and density. The second (moving) mass does not curve the space.

SUMMARY

In the short content of the matrix field theory it is shown that there are <u>two different types</u> of the metric matrix equations and the matrix equations of the field. In the <u>first type</u> all eigenvalues μ_j, $j=1,2,...,n$ of the matrix $g^{-1}R$,

where g is the metric matrix and R is the Ricci matrix, are equal and the appropriate eigenvectors are arbitrary. They are taken equal to the velocity vector. The differential equations of the field are written in the form: $\mu_1 = \mu_2, \quad \mu_1 = \mu_3, \ldots, \mu_1 = \mu_n$.

The metric matrix is defined by these differential equations. In the second type for **only one** eigenvalue μ_j the appropriate eigenvector is equal to the velocity vector,

$$g^{-1} R\, u(\mu_j, x) = \mu_j\, u(\mu_j, x).$$

In the both types we have the equation

$$g^{-1} R\, u(x) = \rho_f\, u(x),$$

where ρ_f is the mass density function, and $u(x)$ is the coordinate velocity.

The solution of the first type metric matrix differential equations in 4-dimensional spacetime with the static spherical symmetry gives the metric matrix with three different constants c_5, c_6, c_7. The interpretation of the constant c_5 is the mass function m_f for the moving mass m., proportional to the moving mass. The interpretation of the constant c_6 is M_f, the mass function for the concentrated mass M, proportional to the concentrated mass, and the interpretation of the constant c_7 is the mass density function $c_7 = -\frac{\rho_f}{3}$. The constants c_6 and c_7 produce the curvature of the space. The constant c_5 does not produce the curvature of the space but influences at the value of the curvature, produced by the concentrated mass and the mass density.

The obtained equations of motion in these metric spaces describe the motion of one mass m in the field of the concentrated mass M and mass density.

REFERENCES

1. Alexander D. Dymnikov, "The matrix theory of the mathematical field and the motion of mathematical points in n-dimensional metric space." Journal of Computational Methods in Sciences and Engineering, 13 (2013) 59-109.

DYSON'S ANALYSIS FOR TOKAMAK GRAVITY WAVE PRODUCTION, LIGO, AND GERTSENSHTEIN COUPLING BETWEEN PHOTONS, GRAVITONS AND MAGNETIC FIELDS

Andrew Beckwith
Chongqing University Department of Physics
Chongqing, PRC, 400044
abeckwith@uh.edu
rwill9955@gmail.com

Received April 20, 2014

Abstract

In a 2013 paper, Freeman Dyson presented thought experiments challenging the detectability of gravitons via LIGO interferometry and via the Gertsheshtein effect. Dyson assumed a distance of several light years would be required for detection of the interaction between gravitational waves (GWs) and tenuous B fields and photons, making gravitons experimentally undectable. In this paper, we present contrary theoretical evidence for detectability of near-field interaction of gravitons, photons, and a magnetic field. Our first example of 100% probability of the Gertshenshtein effect working is due to a GW generated by a Tokamak with a interaction of GW, B field, and photons, in a volume on the order of a few cubic meters. The 100% probability of the Gertshenshtein effect working leads to gravitons interacting with a strong uniform magnetic field, resulting in photons which are detected by appropriate instrumentation and research protocols. We furthermore outline how Dyson was giving misleading information on the efficiency of LIGO, which is inimical to that research initiative on gravitational wave still very pertinent to essential gravitational physics, via the interferometer arrays, which has been noted and corrected

Key words: Gertshenshtein effect, Tokamak, GW, LIGO

Copyright © 2014 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. INTRODUCTION

Dyson in [7] derived criteria as to the probability one could obtain physical phenomenon theoretically modeled by the Gertsenshtein effect [8] . The Gertsenshtein effect [8] is the coupling of magnetic fields, gravitons, and photons. In the Dyson treatment [7] of the Gertsenshtein effect [8] , Dyson hypothesized distances up to many light years for an interaction of magnetic fields, gravitons and photons, for experimental signals which could be detected on the Earth's surface. This assumed geometry of many light years distance lead to the predicted Gertshenshtêin effect [8] unable to allow for graviton detection. In contrast to this assumed vast distances for the Gertshenshtein effect in reference [7], the author has devised via Tokamak generation of gravity waves[3] , which lead to an interaction length of meters for the magnetic field, gravitons, and photons. The reduced length is due to the magnetic field which the gravitons interact with, being inside the detector itself, thereby insuring a 100 % probability for the Gertsenshtein effect occurring. This is commensurate with predictions given in reference [14].The Tokamak example brings up an important point, that even if one wants to measure gravitational waves and detect gravitons from the early universe, that in the 3DSR model for GW detection [22], the Gertshenshtein effect for gravitons, magnetic field, and photons is within the small 3 dimensional geometry of the detector, with an enormous magnetic field. Having the Gerteshenshtein effect [8] in such a small volume dramatically raises the likelihood of detection of gravitons, via resultant photons being picked up by the 3DSR device. Finally , we mention an error in Dyson's argument against LIGO, in which he incorrectly rendered the value of gravitational constant G, times 1 solar mass, divided by the speed of light, squared as equal to about 10 ^ -33 centimeters. The correct value is 1.5 kilometers. It is extremely relevant and very important to keep in mind that interferometric based gravitational wave detectors like LIGO as well as its numerous updates in India and Europe are important also to set up the falsifiable experimental inputs needed to really test gravity theories, as it has been shown very strongly by Dr. Corda in 2009 [5] , and that Dr. Dyson has severely criticized a valuable addition to what is needed to understand gravitational physics. Noting Dr. Dyson's mistake about LIGO [7] is essential for vetting all needed operational tools needed to understand gravity. While Dyson is important, the record needs to be set straight as has been done in this manuscript.

2. Probability for the Gertsentshtein effect, as described by Dyson for the Tokamak GW experiment.

We will briefly report upon Dyson's well written summary results, passing by necessity to the part on the likelihood of the Gertsenshtein effect occurring in a laboratory enviroment[7] . In doing so we put in specific limits as to frequency and the magnetic field, since in our work the objective will be to have at least theoretically a 100% chance of photon-graviton interaction [7] which is the heart of what Dyson reported in his research findings. What we find, is that with a frequency of about 10 to the 9^{th} Hertz and a magnetic field of 10 to the 9^{th} Gauss that there is nearly 100% chance of the Gertsentshtein effect being observed, within the confines of the Tokamak experiment as outlined in [3 , 16] .

In general relativity the metric $g_{ab}(x, t)$ is a set of numbers associated with each point which gives the distance to neighboring points. I.e. general relativity is a classical theory. By necessity, perturbations from flat Euclidian space, are usually configured as ripples in 'flat space', which are the imprint of gravitational waves in space-time. Our paper is to first of all give the probability of a pairing of photons to gravitons linkage, the Gertentshtein effect, as to how the signatures of a perturbation to the metric $g_{ab}(x, t)$ is linkable to photons and vice versa. The Gertentshtein effect is linked to how there is a linkage, signal wise, between gravitons and photons, and we are concerned as to what is a threshold as to insure that GW may be matched to the photons used by Dr. Li and others [7] to signify GW in a detector . To do so let us look at the Dyson criteria as a minimum threshold for the Gertentshtein effect happening [7], namely

$$D \cdot B^2 \cdot \omega \leq 10^{43} \tag{1}$$

The propagation distance is given by ***D***, the magnetic field by ***B***, and the frequency of gravitational radiation is given by ω. We assume that the gravitational frequency is commensurate with the gravitational frequency of gravitons, i.e. that they are, averaged out one and the same thing. In doing so, making use of [7] we suppose on the basis of analysis that *D* is of the order of 10 to the 2nd power, since ***D*** is usually measured in centimeter, and by [7] we are thinking of about a 1 meter If *B* is of the order of 10 to the 9th Gauss Hertz, as deemed likely by [3] , then we have that if the GW frequency , ω is likewise about 10 to the 9th Hertz , that (1) is easy to satisfy. Note that if one has a vastly extended value for ***D***, say 10 to the 13th centimeters that the inequality of (1) does not hold, so that by definition, as explained by Dyson that in a lot of cases, not relevant to[3] , that (1) is not valid, hence there would be no interexchange between gravitons and photons, and hence, if applied to the Dr. Li detector [13, 22] no way to measure gravitons by their photonic signature. Fortunately, as given by [3] this extended version of ***D***, say 10 to the 13th centimeters does not hold. And that then (1) holds. If so then, the probability of the Gertentshtein effect is presentable as, approximately,

$$P \leq \left(10^{36}/B^2 \cdot \omega^2\right) \propto 10^{36}/10^{18} \cdot 10^{18} \sim 1 \equiv 100\% \tag{2}$$

Summing up Eq. (2) is that the chosen values, namely if ***D*** is of the order of 10 to the 2nd power, *B* is of the order of 10 to the 9th power Gauss, and ω is likewise about 10 to the 9th Hertz leads to approximately 100% chance of seeing Gertsenshtein effects in the planned Tokamak experiment in [3]. In making this prediction as to (2), we can say that the left hand side, leading up to the evaluation of P with a numerator equal to 10 to the 36th power will be about unity for the values of *B* detector fields in Gauss (magnetic field) or the generated gravitational field frequency ω from the Tokamak, making an enormous magnetic field in the GW detector itself mandatory, which would necessitate a huge cryogenics effort, with commensurate machinery. Keep in mind that the GW detector is, as given in [3] about five meters above the Tokamak [3] , i.e. presumably the one in Hefei, PRC [16] .

Note, that , ironically, Dyson gets much smaller values of(2) than the above, by postulating GW frequency inputs as to the value of ω about 10 to the 20th Hertz, i.e. our value of ω is likewise about 10 to the 9th Hertz, much lower. If one has such a high frequency, as given by Dyson, the of course, (2) would then be close to zero for the probability of the Gertentshtein effect happening. I.e. our analysis indicates that a medium high GW frequency, presumably close to 10 to the 9th Hertz, and ***D*** 10 to the 2nd power, presenting satisfaction of both (1) and (2). Note the main point though, for large values of ***D***, (1) will not hold, making (2) not relevant, and that means in terms of the Dyson analysis, that far away objects generating gravitons will not be detectable. Via the Gertentshtein effect. There is no such limitation due to a failure of (1) in the Tokamak GW generation setup [3] since then, for Tokamaks, *D* is very small. But if D is large in the case of a lot of astrophysical applications, then almost certainly one never gets to (2) since the Gertsenshtein effect is ruled out. We assume, next that refinements as to the Gertsenshtein effect are in the works, as given by [15,17,18] and next work out a protocol as to the next topic, i.e. early universe shift in space-time geometry leading to GW signals. We will briefly mention what the GW signals are, which are probably accessible if the Gertsenshtein effect is improved upon. Note we will review, briefly, what was given by Weinberg [21] as a black body analysis as to the feasibility of GW/ graviton production via an analysis similar to the black body radiation protocols, and show that the above mentioned figures as to GW/graviton production

3. Brief review of graviton production for massless gravitons, using Weinberg Black body analogy

From the book written by [21].For frequencies, between ω and $\omega + d\omega$: the number of gravitons is given by Weinberg[21], page 287 Formula 10.89 as

$$n(\omega)d\omega = \frac{\omega^2}{\left[\pi^2 \cdot \left(\exp\left[\frac{\hbar \cdot \omega}{k_B T}\right] - 1\right)\right]} \cdot d\omega \tag{3}$$

Integrate this, between two band widths of frequency for the graviton, or for a very narrow graviton frequency width $\bar{\omega}$, the following approximation is acceptable as a modification of (3) as from [21]

$$n(\bar{\bar{\omega}}) \approx \frac{\bar{\omega}^2}{\left[\pi^2 \cdot \left(\exp\left[\frac{\hbar \cdot \bar{\omega}}{k_B T}\right] - 1\right)\right]} \tag{4}$$

Note that k_B above is for the Boltzmann constant, and that T can be set by ANYTHING one wants to have it set by, and the upshot, is that for frequencies approximately as $\bar{\omega}$ approximately of about 10^9 Hz, and with a temperature, of a Plasma as of about 100 KeV, then for (4) figure that one is going to have (4) per unit meter, cubed, in volume, which would lead to variations of easily 10^4 in magnitude from a baseline starting point of say 1 graviton per cubic centimeter, per second. This becomes important when comparing this graviton number, per cubic centimeter against the purported graviton flux number appearing in the case of the Earth given as a Graviton detector which appears in this paper. I.e. see (6) below. The contrast with (4) is stunning.

4 . Why the work by Dyson is not pertinent to long distance approximations as done in his manuscript if the main magnetic field for the Gertsenshtein effect occurs within a detector?

On the face of it, the way the question as to if the Gertsenshtein effect [8] occurs outside a gravitational wave detector appears to be contrived. We assert this is not a contrived question, since the planned detector has a magnetic field many times stronger than what would be expected by conditions on the Earth surface, with Gertsenshtein effects occurring due to the Earth's comparatively very minor magnetic field not playing a role. As given by [8] there is a well defined physical process for graviton-magnetic field interactions which would lead to a photon cascade, enough so, so that large D values, as given above to the tune of many kilometers in length are not advisable or necessary. Needless to say, if one does not believe that the Gertsenshtein effect is not mainly restricted within a GW detector, there are still serious problems with the Dyson formulation.

Review of (1) and (2) above come up with the datum that satisfying (1) is necessary for implementation of (2), i.e. (2) in full generality would likely read as [7]

$$P \sim \sin^2 \sqrt{\left(10^{36}/B^2 \cdot \omega^2\right)} \tag{5}$$

The main absurdity of this formulation is that usually, in interstellar space that one has low B field magnitudes, and low GW frequency values, i.e. ω as low as 100 Hz. Or as high as $\omega \sim 10^9 - 10^{10} Hz$ i.e. in that sense, the Dyson examples chosen as of implementation of (1) and (2) go off the rails, with it being extraordinarily easy for enormous values of $\left(10^{36}/B^2 \cdot \omega^2\right)$ in many situations. I.e. Dyson picked the values of B and also the picked value of $\omega \sim 10^{20} Hz$ is chosen for the purpose of making $P \sim \sin^2 \sqrt{\left(10^{36}/B^2 \cdot \omega^2\right)} \propto 10^{36}/B^2 \cdot \omega^2 << 1$, i.e. Dyson [7] cherry picked the numbers to make the probability for the Gertsenshtein effect as almost non existent, even if (1) were satisfied. But show me

an example where one would have $\omega \sim 10^{20} Hz$ in interstellar space? This is important since $\omega \sim 10^{20} Hz$ is not feasible to entertain in most examples, and if one is looking at GW detectors, as has been done in [3] one is visualizing $\omega \sim 10^{9} - 10^{10} Hz$ in the high end of the GW frequency values, as is given in the Tokamakak example in Section II. I.e. Dyson's analysis [7] of $P \sim \sin^2 \sqrt{\left(10^{36}/B^2 \cdot \omega^2\right)} \propto 10^{36}/B^2 \cdot \omega^2 << 1$ was arbitrarily picked to kill the possibility of a reading of the Gertsenshtein effect [8]. We close this section by asserting that Dyson [7] is confused as to where the Gertsenshtein effect should occur in terms of space-time interactions for proper utilization of a Device physics analysis of where gravitons and B fields interact, and that the large D values he postulates, are not relevant to the case where the Gertsenshtein effect occurs, mainly inside a GW detector. This concludes our analysis of Dyson's failure to properly set up the benchmarks as to analysis of where the Gertsenshtein effect really occurs. So then, we conclude with this statement, and then move to the deficiencies as to Dyson's assertion as to the Earth as a graviton detector, which is section IV below.

5. Dyson's analysis of the Earth as a GW detector. Incomplete physics, and why

We now review the particulars of Dyson's analysis of the Earth as a GW detector [7]. In doing so we are using the same numbers,and our break down of the results show that Dyson is making some assumptions here, which need to be seriously reviewed. In debt with the methodology of finding out what is germane in his analysis to research. To begin with, Dysons, formulae as given in reference [7] which Dyson in his reference calls formula (23) has a next flux of Gravitons hitting the surface of the Earth from the Sun

F(flux)-gravitons hitting Earth = 4×10^{-4} **Gravitons per cm, squared, per second** (6)

In this , using Dysons numbers, he claims that only **1 graviton out of 10 to the 32nd power of gravitons can be detected by the Earth's surface**, assuming a graviton has about a kilovolt of energy i.e. this is, in its heart a situation where Dyson [7] is assuming an absorbtion cross section 10 to the minus 41st power per square centimeter per gram for the Earth, and an absurdly low collision rate. If this were true we are neglecting the Gertsenshtein interaction, since we are assuming no magnetic interface with incoming gravitons. This is only justifiable if there is a hard sphere collision between incoming 'gravitons' and ordinary matter. The analysis is incomplete and unnecessary since Dyson has set up a reseach meme where the Gertsenshtein [7,8] interaction regime stretching kilometers in duration with no fidelity as to the fact that the interaction space between gravitons and a magnetic field is within a GW detector, and does not stretch kilometers in duration away from the GW detector. Having said, that, there is an even

more significant error as to Graviton detection and GW in the Dyson analysis of the LIGO device, which is to be brought up next.

6. Looking at the problem of LIGO , and reviewing Dyson's claims

From [1] there is the following diagram given in their document which we reproduce below as

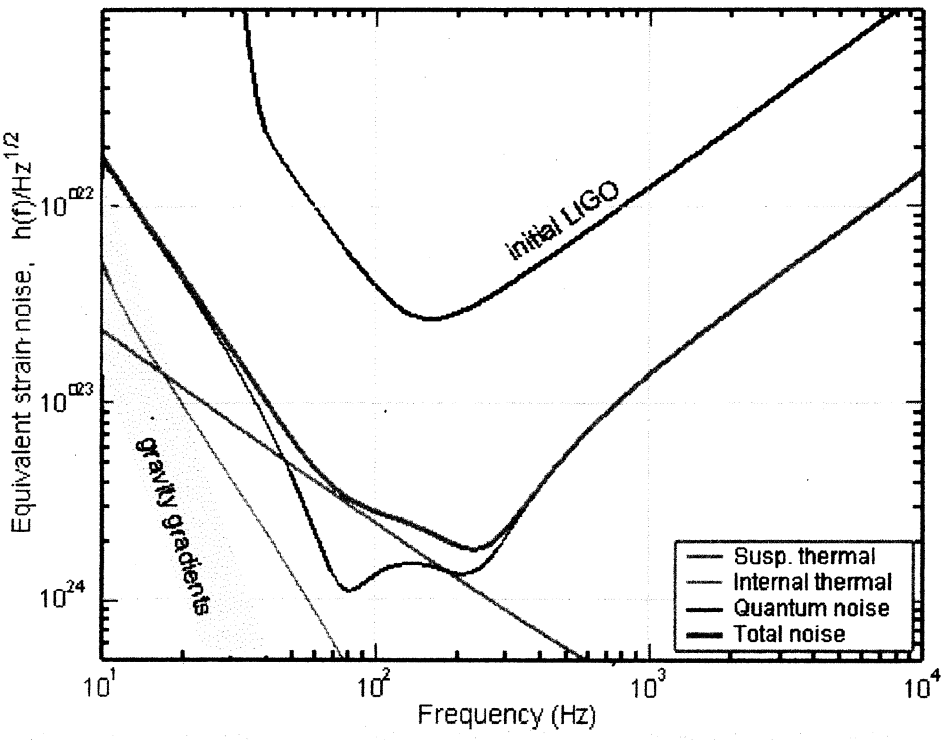


Figure 1 Noise Anatomy of Advanced LIGO. This model of the noise performance is based on the LIGO current requirements set, and represents the principal contributors of the noise and the least-squares sum of those components expressed as an equivalent gravitational wave strain.

From [1] comes the following claim, as given

Futhermore, [1] leads to the following descriptions of detectability, namely as given in its document the following diagram [1] which is reproduced below as

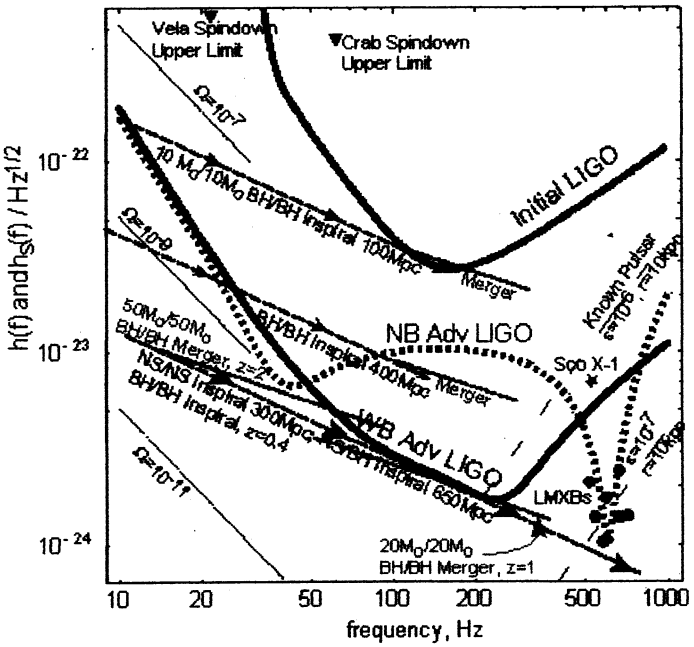


Figure 2 The estimated signal strengths h(f) from various sources (thin lines, filled circles and star) compared with the noise h(f) (heavy lines) of three interferometers: initial LIGO, Advanced LIGO in a wideband (WB) mode, and Advanced LIGO narrowbanded (NB) at 600 Hz. See text for explanations of sources. The signal strength hs(f) is defined in such a way that, wherever a signal point or curve lies above the interferometer's noise curve, the signal, coming from a random direction on the sky and with a random orientation, is detectable with a false alarm probability of less than one per cent using currently understood data analysis algorithms.

The signal strength of LIGO as given by [2] depends upon

$$h \sim \frac{GM}{c^2} \times \frac{1}{r} \times \left[\frac{v}{c}\right]^2 \qquad (7)$$

Here, r is the distance of this gravitational generation from the detector, and v/c is the ratio of say objects within the gravitational detector, and the speed of light. Usually, v/c is much less than 1 , so (7) is particularly relevant to the problem of inspiraling black holes falling into each other, and so, now with this, we should review what Dyson had to say about gravitons, and GW, as well as LIGO .

Right before Dyson's [7], in his section 4, there is a statement that the frequency range for a single graviton to kick an electron out of a single atom, which is 10^{15} Hertz [7]. We will later on comment this estimate [7] as a way to obtain a graviton-photon interaction and also refer to Dyson's claim just before his section5, about thermal graviton generators, that the absorption cross section of ordinary

matter (for a graviton) is 10^{-41} square centimeters per gram. For LIGO, the frequency range is about 10^{2} Hz for two black holes inspiraling into each other, not 10^{15} Hertz, so the option of having a single graviton displace an electron from an atom, is zero. Which leads us to consider the relation given by Dyson, as his [7] formula (10), namely an upper bound to a minimum separation between two objects, say in a LIGO grid, is given by

$$\frac{GM}{c^2} > D \qquad (8)$$

If M is the mass of the sun, then the L.H.S. of (8) is 1.482 times 10 ^ 3 meters, i.e. roughly 1.5 kilometers, or approximately a mile. Assume that then we wish to compare (7) with (8) with a value of V/c ~ 10^ -3, we obtain that two inspiraling black holes with a strain value of h ~ 10^- 22 are about 1000 light years from Earth, for two black holes , combined mass of about one solar mass.

This example in itself, plus Dyson's mathematics should alert the reader, that Dyson, while undoubtedly brilliant in terms of his field theory work and research as up to the 1970s, is not parsing the problem of graviton detection correctly.

Having said this, the next step will be to review what could be done as far as looking at the early universe, as a source of GW, while moving beyond the mistakes we just outlined. In doing so, we assume that if our analysis is complete, we may be able to investigate early universe conditions, via considering if an improvement over the Gertsenshtein effect is possible.

7 . Using the good part of the Dyson analysis, and keeping in mind improvments as to the Gertsenshtein graviton-magnetic field regime are in the offing.

What we have done is to ascertain that the Gertsheshtein interaction is valuable in near field device physics geometry. We have in Section II, where the Dyson analysis can FIX appropriate GW and graviton frequency values, and magnetic field values, so the Gertenshtein interaction is certain to occur. In this, Dyson is warmly thanked for the insight. What we will bring up in closing is that the Gertshenshtein interaction is not necessarily the last word in effective graviton-magnetic field interactions and that improvments are in the offing which could enhance the role of GW detection. To do so, we can make an estimate that from a very simplistic viewpoint, that the view point of what is called the Li effect [13,15,22] involves a magnetic field of the same frequency, direction and

appropriate phase of the gravtional wave field. The Gertsenshtein effect does not involve that E and M field and is proportional to h squared, not h, and in sensitivity the Gertsenshtein effect is about 30 orders of magnitude smaller than the Li effect. For GW of interest. This involves h, which is the strain value of incoming GW entering in a detector.

Eq. (7) and Eq. (8) theoretically could in themselves, if one assumed h ~ 10 ^ - 30, lead to very early universe detection. No one, however, posits that such sensitivity low values could be remotely detectable with conceived of, or extrapolated laser inferometer technology. Also, even in the matter of BHs, entropy speculations, leading to, that the 'entropy' of a BH is given by, where M is the mass of the BH, L_P Planck length, and A_{hor} is the area of the Event Horizon of a black hole., and we state the entropy as [10]

$$S = 4\pi M^2 = \frac{1}{4}\cdot\left(\frac{A_{hor}}{L_P^2}\right) \tag{9}$$

Here, in [10] we have that in its (reference [10]) formulae (24), that its main result is about the differential of the area of an event Horizon which is given as, if there is a Brane theory connection to the formation of BHs, with N the number of dimensions, say up to 10, that what is known as super-radiance , ie. bouncing of incoming radiation off the event horizon is a consequence, of the following derivation, namely if

$$dA_{hor} = \frac{8\pi r_H}{\mathrm{B}} dM_{BH}\cdot\left(1-\frac{1}{\omega}\sum_{j=1}^{N/2} m_j\cdot\Omega_j\right) \tag{10}$$

If dM < 0, then the quantity $\left(1-\frac{1}{\omega}\sum_{j=1}^{N/2} m_j\cdot\Omega_j\right)$ < 0, where the quantum numbers m_j>0 and $\Omega_j = \frac{a_J}{a_j^2 + r_H^2}$ as *frequency of BH arising due to the jth component of BH angular Momentum* J_j as correlated to event horizons of the BH . Such an analysis would have profound effects upon the Dyson analysis of the probability of Graviton detection, where the phenomenon of super-radiance could play a major role as far as GW and gravitons emitted by BHs, especially in the case of inspiraling black holes [10] collapsing upon each other. $a = \sqrt{x^2 + y^2}$ can go to zero, and also $r_H = M_{BH} - \sqrt{{M_{BH}}^2 - a^2}$. Corresponding to BHs with, or without spin, which would affect GW and graviton production.

Having said, that we should examine what could happen if we have a refinement of the Gertsenshtein effect, and its aftermath. Especially as to early universe astronomy

8. Generalization to larger cosmological problems. i.e. what if refinements of the Gertsenshtein effect occur, and allow early universe GW astronomy?

The simplest way to consider what may be involved in alterations of geometry is seen in the fact that in pre-**Octonion** space time regime (which is pre-Planckian), one would have (Crowell, 2005 [6])

$$\left[x_j, x_i\right] \neq 0 \quad \textbf{under ANY circumstances, with low to high temperatures, or flat or curved space.} \tag{11}$$

Whereas in the **Octonion** gravity space time regime where one would have(12) below hold that for enormous temperature increases (Crowell, 2005)[6]

$$\left[x_j, x_i\right] = i \cdot \left[\Theta_{ji}\right] \xrightarrow[Temp \to \infty]{} 0 \tag{12}$$

Here,

$$\Theta_{ji} \sim \Lambda_{NC}^{-2} \sim \left[\Lambda_{4-Dim}\right]^{-2} \propto 1/\left[T^{2\beta}\right] \xrightarrow[T \to \infty]{} 0 \tag{13}$$

Specifically (12) transformed to (13) will undergo physical geometry changes which show up in δ_0 .The space-time shift from pre Planck to the Planck epoch has gravity wave background radiation containing the imprint of the very earliest event. Next, is to consider what happens if Quantum (**Octonion** geometry) conditions hold. The supposition as given by in [12]

Considering all these recent developments, it is plausible that quantum mechanics and gravity has information as a common ingredient, and information is the key to explain the strange connection between two.

When quantum geometry holds, as seen by (14) , GW information is loaded into the **octonion** space time regime, and then transmitted to the present via relic GW which identified via the phase shift in GW as measured in a GW detector. This phase shift is δ_0. The following flow chart is a bridge between the two regimes of (Crowell, 2005)[6] the case where the commutators for QM hold and then again to where the commutators for QM do not hold at all.

$$\left[x_j, p_i\right] \neq -\beta \cdot \left(l_{Planck}/l\right) \cdot \hbar T_{ijk} x_k \xrightarrow[Transition-to-Planckian-regime]{} \left[x_j, p_i\right] = -\beta \cdot \left(l_{Planck}/l\right) \cdot \hbar T_{ijk} x_k \tag{14}$$

(14) above represents the transition from pre-Planckian to Planckian geometry.

Also questions relating to how pre and post Planckian geometries evolve can be answered by a comparison of how entropy, in flat space geometry is linked with quantum mechanics (Lee, 2010)[12] . Once (14) happens, Beckwith hopes to look at the signals in phase shift δ_0

$$\left[x_j, p_i\right] = -\beta \cdot \left(l_{Planck}/l\right) \cdot \hbar T_{ijk} x_k \xrightarrow[Transition-to-release-of-relic-Gravitational-waves-in-flat-space]{} Planckian-Era-Generated-GW \tag{15}$$

Lee's paper [12] gives the details of information theory transfer of information from initially curved space geometry to flat space. When one gets to flat space, then, by (15) one then has a release of relic GW. The readers are referred to appendix A summarizing the relevant aspects of [12] (Lee, 2010) in connecting space time geometry (initially curved space, of low initial degrees of freedom) to Rindler geometry for the flat space regime occurring when degrees of freedom approach a maxima, initially from t > 0s up to about t <1s as outlined in an argument given below in (16). One of the primary results is reconciling the difference in degrees of freedom versus a discussion of dimensions. Also, as Eq. (16) occurs, there will be a buildup in the number of degrees of freedom, from a very low initial level to a higher one, as in the Gaussian mapping [4,13].

$$x_{i+1} = \exp\left[-\tilde{\alpha} \cdot x_i^2\right] + \tilde{\beta} \tag{16}$$

The feed in of temperature from a low level, to a higher level is in the pre Planckian to Planckian thermal energy input as by [4]

$$E_{thermal} \approx \frac{1}{2} k_B T_{temperature} \propto \left[\Omega_0 \breve{T}\right] \sim \tilde{\beta} \tag{17}$$

(17) would have low numbers of degrees of freedom, with an eventual Gauss mapping up to 100 to 1000 degrees of freedom, as described by (Kolb and Turner, 1990)[11] .

It is important to note that the above proposed phase transition is speculative, but it could lead to another source of GW and maybe even Graviton production which with suitable analysis, would lead to more experimental opportunities for astrophysics investigations

Briefly put, this (17) could lead to the other development, namely that in research work as given by [15] (Li, and Yang, 2009), the following case for amplitude

$$A_{\otimes} = A_{\oplus} = \breve{A} \tag{18}$$

Furthermore, first order perturbative terms of an E&M field have its components written as (Li, and Yang, 2009)[15]

$$\tilde{F}^{(1)}_{0\,2} = i\tilde{F}^{(1)}_{0\,1} \tag{19}$$

Secondly, there is a way to represent the" number" of transverse first order perturbative photon flux density as given in an earth bound high frequency GW detector [15] .

$$n_r^{(1)} = \frac{c}{2\mu_0 \hbar \omega_{e^-}} \mathrm{Re}\{\ \} \tag{20}$$

$$\{\ \} = i\left(\exp[-i\theta]\right) \cdot \tilde{F}^{(1)*}_{0\,1} \cdot \left[\frac{i}{\omega_{e^-}} \cdot \left(\frac{\partial \Psi_x}{\partial y} - \frac{\partial \Psi_y}{\partial x}\right)\right] \tag{21}$$

Here the quantity $\frac{i}{\omega_e}\cdot\left(\frac{\partial \Psi_x}{\partial y}-\frac{\partial \Psi_y}{\partial x}\right)$ represents the z component of the magnetic field of a Gaussian beam used in an EM cavity to detect GW. We introduce the quantity Q, the quality factor of the detector cavity set up to observe GW, and $\breve{A}$, the experimental GW amplitude. In the simplest case, $\hat{B}_y^{(0)}$ is a static magnetic field. Then $\tilde{F}_{0\ 2}^{(1)} = i\tilde{F}_{0\ 1}^{(1)}$ leads to by [15]

$$\tilde{F}_{0\ 1}^{(1)} = i2\breve{A}\hat{B}_y^{(0)}Q\cdot\left[\sin\left[\frac{n\pi z}{b}\right]\right]\cdot\exp\left[i\left(-\omega_g t+\delta_0\right)\right] \tag{22}$$

The formula $E_{thermal}\approx\frac{1}{2}k_B T_{temperature}\propto\tilde{\beta}$ [13] is a feed into ω_g provided time $t\propto$ Planck time, and set (22) with $\omega_g\sim\omega_g$ by setting up $E_{thermal}\approx\frac{1}{2}k_B T_{temperature}\approx\tilde{\beta}$. In other words, for relic GW production, a interrelationship between $\tilde{\alpha}$ and $E_{thermal}\approx\frac{1}{2}k_B T_{temperature}\propto\tilde{\beta}$ for increases in degrees of freedom. This is a different perspective than what is normally used in analyzing what happens in a transition between initial Planck time ~ 10^{-44} seconds, and cosmological evolution up to 10^{-30} seconds The next discussion is on research done by [14], as to identifying traces of massive gravitons

8a. Re casting the problem of detecting Gravitons in a detector for "massive" Gravitons

We now turn to the problem of detection. The following discussion is based upon with the work of Dr. Li, Dr. Beckwith, and other physics researchers in Chongqing University [14].. What (Li et al, 2003) have shown in 2003 [14] which Beckwith made an extension is to obtain a way to present first order perturbative electromagnetic power flux, i.e. $\overset{(1)}{T}^{\mu\nu}$ in terms of a non zero four dimensional graviton rest mass, in a detector , in the presence of uniform magnetic field [14] .What if we have curved space time with an energy momentum tensor of the electro magnetic fields in GW fields as given by [14] ?

$$T^{\mu\nu}=\frac{1}{\mu_0}\cdot\left[-F_\alpha^{\mu}F^{\nu\alpha}+\frac{1}{4}\cdot g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right] \tag{23}$$

By reference [14] we state that $F_{\mu\nu}=F_{\mu\nu}^{(0)}+\tilde{F}_{\mu\nu}^{(1)}$, with $\left|\tilde{F}_{\mu\nu}^{(1)}\right|<<\left|F_{\mu\nu}^{(0)}\right|$ will lead to

$$T^{\mu\nu}=\overset{(0)}{T}^{\mu\nu}+\overset{(1)}{T}^{\mu\nu}+\overset{(2)}{T}^{\mu\nu} \tag{24}$$

The 1st term to the right side of (24) is the energy – momentum tensor of the back ground electro magnetic field, and the 2nd term to the right hand side of (24) is the first order perturbation of an electro magnetic field due to the presence of gravitational waves helps constrain (25) just below. This discussion as to section 8 is admittedly very preliminary, but it could be a way forward as to beginning to use the concept of a 'current' as in a GW/graviton detector, which with much more detail could take into account early universe phase transitions which occur at the beginning of the inflationary era. Secondly in conjunction with reference [20], it may remove problems associated with heavy gravity.

$$J_{effective} \cong n_{count} \cdot m_{4-D-Graviton} \tag{25}$$

As stated, [9,11] $m_{4-D-Graviton} \sim 10^{-65} grams$, while n_{count} is the number of gravitons which may be in the detector sample. What Beckwith and Li intend to do is to isolate out an $\overset{(1)}{T}^{\mu\nu}$ assuming a non zero graviton rest mass. . I.e. use $\tilde{\beta} \cong |F|$ and make a linkage with $\overset{(1)}{T}^{00}$. The term $\overset{(1)}{T}^{00}$ isolated out from $\overset{(1)}{T}^{\mu\nu}$. The point is that detected GW

9. Conclusion. Much work needs to be done, including refinement of the Gerstsenshtein effect, and analysis of where GW /graviton production is investigated for astrophysical processes.

This paper raises questions as to the appropriateness of the Dyson analysis, in particular the Dyson dismissal of LIGO is based upon an incomplete rendering of a distance, D, as less than Planck Length, which we disprove by elementary analysis of the left hand side of (8) which with one solar mass is 1.48 kilometers, 1 mile, in value, as opposed to the Dyson sub Planck length. It is worth noting that LIGO has kilometer long interferometer arms, and plenty of space, as to the obtaining GW and/or Graviton itself in instrumentations. Dyson [7] also insisted upon evaluation of the Gertsheshtein effect in terms of light year distances as to light and magnetic field interactions, thereby concluding with virtually non existent Graviton interaction with instrumentation. For one thing, as given in the early part of the manuscript, what Dyson hypothesized for the probability of Gertshenshtein interaction for measurable gravitons as to a Tokamak generation of GW is appropriate and may be , for sufficiently large strain values of h~ 10^-25, may be detected with advanced instrumentation. The problem is this. What Dyson postulates as to the probability of a Gertsenshtein interaction between Gravitons and a magnetic field is no issue in that situation. I.e. a very strong magnetic field would be inside the detector itself.

The Tokamak discussion is the opposite situation from the vast distances Dyson postulated photons traveled versus intervening galactic magnetic fields, as then producing gravitons, is actually the reverse of the situation expected and modeled by Dr. Li and others [13 ,15, 22] I.e. the Gertenshtein effect is for within a DETECTOR device, and Dyson's calculations [7] as to light year distance of traveling of photons through magnetic fields is the reverse of the situation which was designed by the American and Chinese teams using 3DSR technology[22].

Dyson's analysis is in several specific cases not related to the actual situation of GW/ Graviton detection. As an example, Dyson states that 10^{15} Hz for a graviton is required as to kicking an electron out of an atom [7], as though such a frequency is what would be expected of gravitons/GW. The fact is, that the Gertshenshtein effect does not need a frequency of 10^{15} Hz due to GW / gravitons, to lead to detectable signals, in a detector.

At the same time, the directness of the questions asked by Dyson is welcome and the author acknowledges that until Dyson framed his article questions, that much of the GW/graviton issues were too incompletely rendered to permit an analysis of the relevant experimental issues.

Acknowledgement

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

References

[1] http://www.ligo.caltech.edu/advLIGO/scripts/ref_des.shtml

[2] http://www.tapir.caltech.edu/~teviet/Waves/gwave.html

[3]Andrew Beckwith," Review of the Grischuk and Sachin Gravitational Wave Generator Via Tokamak Physics", http://vixra.org/abs/1311.0132

[4]Andrew Beckwith, How to use the cosmological Schwinger principle for energy flux, entropy, and "atoms of space–time" to create a thermodynamic space–time and multiverse". DICE 2010 (Pisa, Italy)contributed talk, 2011 *J. Phys.: Conf. Ser.* **306 012064** doi:10.1088/1742-6596/306/1/012064 ; http://iopscience.iop.org/1742-6596/306/1/012064

[5] C. Corda "Interferometric detection of gravitational waves: the definitive test for General Relativity", Int. J. Mod. Phys. D 18, 2275 (2009)

[6] L. Crowell, *Quantum Fluctuations of Space-time*, in World Scientific Series in Contemporary Chemical Physics, Volume 25, Singapore, Republic of Singapore, 2005

[7]FREEMAN DYSON, *Int. J. Mod. Phys. A*, **28**, 1330041 (2013) [14 pages] DOI: 10.1142/S0217751X1330041X

[8] M. E. Gertsenshtein, 1961. Wave Resonance of Light and Gravitational Waves", JETP, 41, 113-114, English translation in Soviet Physics JETP, 14, 84-85 (1962)

[9] A. S. Goldhaber, M.M. Nieta "Photon and Graviton Mass Limits", http://arxiv.org/pdf/0809.1003.pdf Arxiv 0809.1003 V 5 [hep-th] 4 October 2013

[10] E. Jung,S.Kim,D.K. Park, "Conditions for the Super-radiance Mode in Higer Dimensional Rotating Black Holes with Angular Momentum Parameters",Phys.Lett.B.619,(2005), pp. 345-351, Arxiv:hep-th/0504139

[11] E. Kolb, E. and S. Turner, *The Early Universe*, Westview Press, Chicago, USA, 1991

[12] J.W. Lee, "Quantum Mechanics Emerges from Information Theory Applied to Causal Horizons", Found Phys, DOI: 10.1007/s10701-010-9514-3, pp 1-10 (on line November 3, 2010)

[13] F Y Li et al. Eur. Phys. J. C 56, 407-423 (2008) [http://www.gravwave.com/docs/Li-Baker%206-22-08.pdf

[14] F. Li,M . Tang, D. Shi. "Electromagnetic response of a Gaussian beam to high frequency relic gravitational waves in quintessential inflationary models", PRD 67,104008 (2003), pp1-17

[15] F. Li and N. Yang, " Phase and Polarization State of High Frequency Gravitational waves", *Chin Phys. Lett. Vol 236, No 5(2009), 050402, pp 1-4*

[16] H.. Li,. Y. Guo, et.al.," A long-pulse high-confinement plasma regime in the Experimental Advanced Superconducting Tokamak", *Nature Physics* (2013) doi:10.1038/nphys2795; Received 17 April 2013 Accepted 24 September 2013 Published online 17 November 2013

[17] M. Maggiore, *Gravitational Waves: Volume 1. Theory and Practice* Oxford University press, 2008, Oxford UK

[18] M. Meyers *Meyers – Perry Black holes*, pp 101-131 in *Black holes in higher dimensions* edited by G. Horowitz, Cambridge University Press, NYC, NY, USA, 2012

[19] T. Padmanabhan *Gravitation, Foundations and Frontiers* Cambridge University Press, NYC, NY, USA 2010

[20] P. Tinyakov,"Giving Mass to the Graviton", course 12, pp 471-498, of Session LXXXVI, *Particle Physics and Cosmology, the fabric of Space – Time*,editor F. Bernardeau, C. Grojean, J. Dalibard, 2006 (Les Houches), Elsevier , Oxford, UK

[21] Steven Weinberg, ***Gravitation and Cosmology, Principles and applications of the General theory of Relativity***, John Wiley Sons, 1972.

[22] R. Clive Woods, Robert M L Baker, Fangyu Li, Gary V Stephenson, Eric W Davis, Andrew W Beckwith; **A New Theoretical Technique for the Measurement of High-Frequency Relic Gravitational Waves**, Journal of Modern Physics, SCIRP, **DF** PP. 498 - 518 **DOI:** 10.4236/jmp.2011.26060

COMMENT ON "WHAT THE INFORMATION LOSS IS NOT"

Qing-yu Cai, Baocheng Zhang, Miangsheng Zhan
Wuhan Institute of Physics and Mathematics
Chinese Academy of Sciences, Wuhan 430071
People's Republic of China
qycai@wipm.ac.cn

Li You
Department of Physics, Tsinghua University
People's Republic of China
lyou@mail.tsinghua.edu.cn

Received April 4, 2014

Abstract

A recent article by Mathur attempts a "precise formulation" for the paradox of black hole information loss [S. D. Mathur, arXiv:1108.0302v2 (hep-th)]. We point out that a key component of the above work, which refers to entangled pairs inside and outside of the horizon and their associated entropy gain or information loss during black hole evaporation, is a presumptuous false outcome not backed by the very foundation of physics. The very foundation of Mathur's above work is thus incorrect. We further show that within the frame work of Hawking radiation as tunneling the so-called small corrections are sufficient to resolve the information loss problem.

Keywords: Information loss paradox, entropy conservation, small correction, pure state, entanglement.

Copyright © 2014 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1 The black hole information loss paradox

The black hole information loss paradox can be appreciated without getting into the details of quantum gravity. As first addressed by Hawking [1], the information loss paradox, or the Hawking paradox, refers to the ever increasing entropy of the thermal radiations (Hawking radiations) from a black hole. If the radiations are thermal (thermally distributed) with no correlations between individual radiated quanta, an increasing entropy equals to loss of information. In order to resolve the paradox, one must find a mechanism capable of balance the entropy increase. Many scenarios for possible resolutions have been discussed at varying levels of details. For instance, we recently point out the increasing entropy is balanced exactly by the nontrivial correlations among Hawking radiations if the non-thermal spectrum based on Hawking radiation as tunneling is adopted.

The information-carrying correlation we discover will play a crucial role in any resolution. Yet, the Hawking paradox cannot be considered resolved even if these correlations are detected. A complete resolution can only occur after entropy conservation is confirmed experimentally, which then implies the dynamics of Hawking radiation is consistent with unitarity as required by quantum mechanics, despite our inabilities in writing down a time-dependent wave function for the emissions consistent with quantum statistics and general relativity.

2 Mathur's "precise formulation" of the paradox

Recently, Mathur concludes that small corrections cannot resolve the information loss paradox [2]. His reasoning rests upon the so-called "precise formulation" of the paradox, outlined in four steps A-D [3]. In his step (C) "Creation of the entangled state", EPR pairs of the type

$$|\Psi\rangle_{\text{pair}} = \frac{1}{\sqrt{2}}(|0\rangle_{c_i}|0\rangle_{b_i} + |1\rangle_{c_i}|1\rangle_{b_i}), \tag{1}$$

are assumed to be created one after another near the horizon with evolution. After N pairs the whole wave function then takes the following form

$$|\Psi\rangle \approx |\Psi\rangle_M \otimes (\frac{1}{\sqrt{2}}|0\rangle_{c_i}|0\rangle_{b_i} + \frac{1}{\sqrt{2}}|1\rangle_{c_i}|1\rangle_{b_i})^{\otimes N}, \tag{2}$$

where $|\Psi\rangle_M$ is the state of the matter shell. The regions outside the horizon are labelled b_i while those inside the horizon are labelled c_i. the numbers 0 and 1 denote the occupation number of a particular mode. The mass of a black hole decreases by the ingoing (negative massed) particle c_i with each (positive massed) particle b_i outside the horizon becoming Hawking radiation. In the end, the hole vanishes due to exhaustion.

Mathur concludes the initial pure state $|\Psi\rangle_M$ for the infalling hole evolves into a mixture claiming it is not entangled with emissions b_i, outside the horizon. b_i is therefore left also in a mixed state as there is nothing remaining in the hole for it to be entangled with. This is clearly erroneous as we show in detail below. When the initial correlation between c_i and b_i is considered as Mathur does in his 'precise formulation', b_i actually evolves into a pure state upon the annihilation of c_i into the black hole. Rather than relying on unstated assumptions or philosophical arguments, we will walk through Mathur's scenario line by line, step by step, exam his mistake in detail.

Let us consider the EPR pair state $|\Psi\rangle_{\text{pair}} = (|0\rangle_{c_i}|0\rangle_{b_i} + |1\rangle_{c_i}|1\rangle_{b_i})/\sqrt{2}$ for the particles b_i and c_i respectively outside and inside the horizon. The state for particle b_i is found to be $\rho_{b_i} = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$ after tracing out c_i, which is a mixed state, actually a maximally mixed state. Likewise, we find $\rho_{c_i} = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$. The respective entropies for the b_i and c_i are

$$S(b_i) = S(c_i) = \ln 2, \tag{3}$$

which are nonzero as both are in mixed states. However, the EPR pair state $|\Psi\rangle_{\text{pair}}$ is a pure state, whose entropy is simply zero, or $S(|\Psi\rangle\langle\Psi|_{\text{pair}}) = 0$. This helps us to obtain the entropy of the correlation between b_i and c_i

$$S(b_i : c_i) = -2\ln 2, \tag{4}$$

as $S(b_i) + S(c_i) + S(b_i : c_i) = 0$.

When a negative energy (mass $m_{c_i} = -\omega$) particle c_i falls into the hole, it eventually annihilates a positive energy matter a_i of $m_{a_i} = \omega$. The mass of the hole thus decreases by $-\omega$ since $m_{a_i} + m_{c_i} = 0$. The annihilation actually causes the particles a_i and c_i to end up in a maximally entangled state, a point presumptuously skipped over in Mathur's reformulation. This is the reason for his subsequent false statement and conclusion. In essence, the annihilation is like a joint measurement in teleportation, causing the state of particle a_i to be teleported to the emission b_i

outside the horizon [4]. If a_i initially exists in a pure state, b_i will end up in a pure state, not a mixed state as Mathur concluded, although as we show below b_i is no longer entangled with c_i or a_i after their annihilation inside the horizon.

To be more precise, the joint state $|\Psi\rangle$ for the whole black hole including EPR pairs cannot be written down in a product state Eq. (2) as Mathur used because its temperature changes after each emission ω, *i.e.*, $M \to M - \omega_i$. Taking this into account, the joint state is more appropriately expressed as

$$|\Psi\rangle = \lambda_i |a_i\rangle |c_i\rangle |b_i\rangle, \tag{5}$$

where $\lambda_i \sim e^{-4\pi M_i \omega_i}$ denotes the temperature change due to an emission b_i. When the hole is exhausted due to annihilations of a_i and c_i pairs, the initial pure state of the hole, $|\Psi\rangle_M$, is teleported to the Harking radiations. A detailed discussion of the above is already present and can be found in Ref. [9], where one finds the final state of Hawking radiation is a pure state [see Eq. (9) of [9]], not a mixed state as assumed by Mathur.

We thus claim Mathur's conclusion that the initial pure state $|\Psi\rangle_M$ will involve into a mixed state because the quanta b_i outside the horizon is in a mixed state is false.

3 Small corrections are sufficient

The temperature of a black hole is inversely proportional to its mass, thus it is most easily described in terms of an ensemble with its entropy, $S_{\rm BH} = 4\pi M^2$, or the Bekenstein entropy for a Schwarzschild black hole [10]. Although it is possible in principle to carry out a pure state analysis of the Hawking radiation dynamics, we do not know how to write down an appropriate initial pure state. The statistical properties of a black hole, on the other hand, can be described once its entropy is known. After the hole is vanished into Hawking radiations, the entropy for the remaining system made up of emissions becomes $S_{\rm emissions} = \sum_i S(\omega_i)$ with $M = \sum_i \omega_i$ due to energy conservation. The information loss paradox arises as a result of total entropy increase, or $S_{\rm emissions} > S_{\rm BH}$, if the spectrum for Hawking radiation is thermal.

Based on an earlier work by us [6, 7], it is straightforward working backwards to find out what kind of spectrum for Hawking radiation is needed in order to resolve the paradox making use of the information carrying correlations between emissions. This

rests in our belief of unitarity in quantum mechanics, *i.e.*, information is conserved, which requires the total entropy of a closed system, composed of the hole and the emissions in the present case, to be conserved. After an emission of energy ω, the entropy for the black hole decreases by $\Delta S_{\rm BH} = 4\pi(M-\omega)^2 - 4\pi M^2 = -8\pi M\omega + 4\pi\omega^2$. From $\Delta S_{\rm BH} = \ln\Gamma(\omega)$, we find the appropriate spectrum is $\Gamma(\omega) = \exp[-8\pi\omega(M-\omega/2)]$, or precisely the nonthermal spectrum of Parikh and Wilczek [5]. When compared to Hawking's thermal spectrum, it contains a small correction $O(\omega^2)$, due to the back action of the quantum emission event. Although tiny in magnitude for most massive black holes, this small correction in the Parikh-Wilczek spectrum [5] provides exactly what is needed to balance the total entropy increase with information contained in correlations among Hawking radiations. Thus, due to our discovery of the non-trivial correlations among radiations governed by the Parikh-Wilczek spectrum [6], one can now conclude that small corrections in the nonthermal spectrum are sufficient to resolve the paradox of black hole information loss.

4 Mathur's criticism on our resolution

As outlined above, the spectrum of Parikh and Wilczek, based on Hawking radiation as tunneling, is sufficient to resolve the information loss paradox. In the article by Mathur [3], however, a criticism is raised regarding our resolution (see page 14 of Ref. [3]).

Mathur argues that there exists a missing prefactor in the Parikh-Wilczek spectrum $\Gamma(\omega) = \exp[-8\pi\omega(M-\omega/2)]$, which should have the appropriate dimension of frequency like an attempt frequency in the standard tunneling treatment for a particle of mass ω. Thus $\Gamma(\omega)$ represents the probability flux of transmitted particles. This point of a missing prefactor is certainly correct. Generally speaking, $\Gamma = \Lambda\exp[-8\pi\omega(M-\omega/2)]$ indeed comes with a prefactor Λ. When the probability flux of the incident particles is assumed unity, the prefactor becomes $\Lambda = 1$.

The key criticism of Mathur to our resolution concerns the crucial relationship used by us for the nonthermal spectrum,

$$\Gamma(\omega_1,\omega_2) = \Gamma(\omega_1)\Gamma(\omega_2|\omega_1) = \Gamma(\omega_1+\omega_2). \tag{6}$$

Mathur proclaims this to be true even for the thermal spectrum $\Gamma_T(\omega) = \exp(-8\pi M\omega)$, which is a totaly erroneous as we show below. Using statistical theory, it is easy to find for the thermal spectrum the various terms are given by $\Gamma(\omega_2|\omega_1) = \exp[-8\pi(M-$

$\omega_1)\omega_2]$, $\Gamma(\omega_1,\omega_2) = \Gamma(\omega_1)\Gamma(\omega_2|\omega_1) = \exp[-8\pi M(\omega_1+\omega_2)]\exp(8\pi\omega_1\omega_2)$, and $\Gamma(\omega_1 + \omega_2) = \exp[-8\pi M(\omega_1+\omega_2)]$. Thus it is easy to check that $\Gamma(\omega_1,\omega_2) = \Gamma(\omega_1 + \omega_2)\exp(8\pi\omega_1\omega_2) \neq \Gamma(\omega_1+\omega_2)$. Therefore, this key criticism from Mathur to our resolution is based on a rather elementary mistake in his calculation of the conditional probability and joint probability. His criticism cannot stand.

Finally, we stress that the essential difference between the thermal spectrum of Hawking and the nonthermal spectrum of Parikh-Wilczek translates into the absence and existence of correlations among different emissions. Although rather small between any two individual emissions, the correlations among all pairs of Hawking radiations have the capacity to carry off all information in the initial black hole, thus the information loss paradox is resolved [8].

5 Conclusion

In summary, we point out that Mathur's "precise formulation" of the information loss paradox lacks content and is false in several of the key components. His key calculations are incorrect and his conclusion regarding our resolution based on Hawking radiation as tunneling is erroneous. In the simply example he provides to discuss entanglement and correlation between two particles in an EPR pair respectively residing inside and outside the horizon, the final state of emissions is a pure state when quantum correlations are properly treated, again in stark contrast to his presumptuous statement of a mixture state.

We further point out that Mathur's criticism on our resolution of Hawking radiation as tunneling is once again due to an incorrect use of probability theory. We show using straightforward statistics and probability theory that there exists information-carrying correlations among Hawking radiations [6, 7, 8] and the entropy is conserved during Hawking radiation process, when the non-thermal spectrum of Parikh and Wilczek is assumed, or when back reaction from Hawking radiation is considered [5]. Our discovery thus proves undoubtedly that no information is lost in Hawking radiation. The small correction in the spectrum of Parikh and Wilczek is necessary and sufficient to resolve the information loss paradox [6, 7, 8].

References

[1] Hawking S., Breakdown of predictability in gravitational collapse. *Phys. Rev. D* **14**, 2460. (1976).

[2] Mathur, S. D. The information paradox and the infall problem. *Class. Quantum Grav.* **28**, 125010 (2011); arXiv:1012.2101 (hep-th).

[3] Mathur, S. D. What the information paradox is *not.* arXiv:1108.0302v2 (hep-th).

[4] Charles H. Bennett, Brassard G., Crépeau C., Jozsa R., Peres A., and Wootters W. K., Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **70**, 1895 (1993).

[5] Parikh M. K. & Wilczek, F. Hawking radiation as tunneling. *Phys. Rev. Lett.* **85**, 5042 (2000).

[6] Zhang, B., Cai, Q. Y., You, L., & Zhan, M. S. Hidden Messenger Revealed in Hawking Radiation: a Resolution to the Paradox of Black Hole Information Loss. *Phys. Lett. B* **675**, 98 (2009).

[7] Zhang, B., Cai, Q. Y., Zhan, M. S., & You, L. Entropy is conserved in Hawking radiation as tunneling: A revisit of the black hole information loss paradox. *Ann. Phys. (N.Y.)* **326**, 350 (2011).

[8] Israel, W, and Yun, Z. Band-aid for information loss from black holes. *Phys. Rev. D* **82**, 124036 (2010).

[9] Lloyd S., Almost Certain Escape from Black Holes in Final State Projection Models. *Phys. Rev. Lett.* **96**, 061302 (2006).

[10] Bekenstein J. D., Black Holes and Entropy. *Phys. Rev. D* **7**, 2333 (1976).

THE SUBSTRUCTURE OF THE QUANTUM FIELD-OSCILLATOR

Guido Giovanni
Department of Mathematics and Physics
High School "C. Cavalleri"
Via Spangliardi 23
Parabiago, 20015 Italy
liceocavalleri@libero.it

Received March 30, 2014

Abstract

An oscillator (IQO) more elementary than the quantum one is proposed. The IQO expresses the quantum operators (a, a^+) with two-components, and it is composed by two sub-oscillators, each with "semi-quantum" ($\frac{1}{2}\hbar\omega$). The commutation relation of the (a,a^+) "semi-quanta" is then calculated, and the structure of scalar fields in terms of IQO is defined, with and without electric charge. A coupling mechanism of electrically neutral IQO's is analyzed, that produces a pair of IQO's (matter-antimatter) with opposite electric charges.
By IQO it is moreover formulated a hypothesis on the internal structure of a quark, calculating its non-integer value of the electric charge (Q), interpreting the isospin (T) as the internal current of semi-quanta, and hypercharge (Y) as a descriptive variable of such a structure. Finally, we conjecture the geometric structure of the IQO's constituting quarks (u, d) and proton, further calculating successfully the ratio (m_d/m_u) of their masses.

Keywords: IQO; semi-quantum; semi-quanta; sub-oscillator; commutation relation; electric charge; quark; isospin; hypercharge; geometric structure.

1 Introduction

In this study we note that the quantum oscillator, representative of a quantum field, differs from the classical one, because it expresses the possibility of distinguishing the "direction" of the phase rotation associated with the oscillation. We show that the sign of the electric charge of a field-particle can be derived from the existence of a degree of freedom inside the oscillator, which is connected to the direction of phase rotation. To such a purpose we proceed to the study of the quantum oscillator in an original way.

Copyright © 2014 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

At the beginning **(sect. 1)**, it is remarked that any type of oscillator (as the one harmonic one, as the one driven-damped, or as the one elastically coupled to other oscillators) represents a solution of the motion equations in terms of two oscillating components, defined as elastic and inertial component.
This leads us to build a two-component representation for the quantum field-oscillator.
It follows that each of the two operators of annihilation (**a**) and creation ($\mathbf{a^+}$) is expressed by two components: elastic and inertial

$$[\mathbf{a} \rightarrow (\mathbf{a_{el}}, \mathbf{a_{in}}) ; \mathbf{a^+} \rightarrow (\mathbf{a^+_{el}}, \mathbf{a^+_{in}})].$$

This original form of quantum oscillators allows to define a more elementary structure, consisting in two coupled sub-oscillators which overall contain two "semi-quanta" (with energy ($\frac{1}{2}\hbar\omega$)) described by "semi-quanta" operators.
The oscillator with a semi-quantum structure is denoted by **"IQO"** (the acronym of **I**ntrinsic **Q**uantum **O**scillator), stating that it can represent very well a Φ-scalar quantum field (IQO-field oscillator).
In **Sect. 2** a new exponential form of the operators ($\mathbf{a}$, $\mathbf{a^+}$) [1] is proposed, which allows to determine the mathematical form of the "semi-quanta" operators, as well as their new commutation brackets, and, finally, the distribution of semi-quanta in two sub-oscillators.
Taking into account both the distribution of semi-quanta inside the two sub-oscillators of an IQO and their movement in relation to the phase (expressed by an operator (**r**) of phase rotation), one may infer that a new degree of freedom is possibly determined in a quantum oscillator with semi-quanta. This degree of freedom inside the quantum field-oscillator might allow to detect the direction of the phase rotation, which would then be interpreted as the sign of the electric charge of a particle.
Subsequently, in **Sect. 3**, we formulate a coupling mechanism between two real scalar fields Φ-IQO's [indicated with $\Phi_1 \oplus \Phi_2$], which leads to the final formation of a quantum system consisting of two entangled complex scalar fields Ψ-IQO's, known in the literature as an electrically charged solutions to the KG (Klein-Gordon) equation.
In sect. 4, using the structure by semi-quanta of the quantum oscillator, the values of the electric charge Q are calculated, expressed now by the "pair" of IQO fields (Ψ_+,Ψ_-) which are not entangled.
It shows that any irregular structure of semi-quanta Ψ-IQO's can be "regularized" by an additional elastic coupling with a neutral Φ-IQO.
Furthermore, it is shown that the $[\Phi_1 \oplus \Phi_2]$ coupling of two Φ-IQO's with opposite eigenvalues of the operator (**r**), gives rise to a new pair of IQO-fields (Ψ_1, Ψ_2) with opposite electric charge, in which each one has semi-quanta operators with the same phase-rotation direction.
One notes that : $(\Psi_1 , \Psi_2) \equiv [\Psi_+ + \Psi_-]$.
The coupling process $[\Phi_1 \oplus \Phi_2] \rightarrow [\Psi_+ + \Psi_-]$ reminds us of the well-known creation of pairs, described in quantum field theory.
We think that the coupling mechanism between two or more IQO-particles can be essential to build up the different fields and their interactions.

In **section 5**, taking up the idea of "internal" rotation of the phase, further "internal" rotations into a "composed" particle (as an hadron) are devised. In fact, taking in consideration that hadrons are particles composed by quarks and that they have a multiplet structure, we find that the semi-quantum structure points at a particular *space* inside each elementary particle (even quarks), if we associate to phase, to isospin, and to spin some internal rotations, of which electrical charge (Q) and isospin (T) are the generators.
The hypothesis of an "internal space" becomes more appropriate if one interprets the isospin (T) as a physical variable describing an internal current of quark semi-quanta, and the hypercharge Y [formulated by Gell-Mann to describe the variety of hadron multiplets (mesons and baryons)] as the index of an internal structure (**hypothesis of structure**) of IQO-Quarks (in hadrons), or of Sub-IQO (within a single quark)! Observing the close connections between Q, T and Y, we construct a matrix calculation of the electric charge. By using the coupling mechanism described in Section 3, we formulate the form of the representative matrix of a quark. Then on using the matrix calculation of Q, the values are determined of the electric charge of a IQO-quark. This non-integer values are identical to those assigned to precisely one quark; this suggests the non-integer value of charge as indicating probability to detect the representative quantum of a quark (so to say, by a photon explorer).
The probabilistic interpretation of the value of the electric charge justifies the no detection of an isolated quark, without the need of the concept of quark confinement.
Finally, **in Sec. 6**, we conjecture a geometric structure of quarks (u, d) (that of a triangle of couplings of internal sub-IQO's), and define the geometric structure of the proton. Given the structure of "golden" triangles, it follows the ratio of the mass of quarks (u, d).

1 The operators (a, a^+) of quantum oscillator with two component

Be the classical oscillator in the (q, p) coordinates of the phase plane coordinates; its Hamiltonian is

$$H=\left(\frac{p^2}{2m}\right)+\left(\frac{m\omega_0^2 q^2}{2}\right) \tag{1}$$

With equations of motion

$$\begin{cases}\dfrac{dp}{dt}=-m\omega_0^2 q\\ \dfrac{dq}{dt}=\left(\dfrac{p}{m}\right)\end{cases} \tag{2}$$

Its general solutions are linear combinations of "two" independent oscillations

$$\left\{ \begin{array}{l} q(t) = q(0)\cos(\omega_0 t) + \left(\frac{p(0)}{m\omega_0}\right)\sin(\omega_0 t) \\ p(t) = p(0)\cos(\omega_0 t) - (m\omega_0)q(0)\sin(\omega_0 t) \end{array} \right\} \quad (3)$$

Instead, in damped oscillator ($\Gamma \neq 0$), the general solution is

$$\left(q(t)_{damp}\right) = \left(e^{-(1/2)\Gamma t}\right)\left[A_s \sin(\omega_s t) + B_s \cos(\omega_s t)\right] \quad (4)$$

Where $\omega_s \neq \omega_0$, and $q(t)_{damp}$ is any coordinate.
Note that there are two terms. The energy of the oscillator is:

$$E(t) = E_0\left(e^{-\Gamma t}\right) = \left(e^{-\Gamma t}\right)\left[\frac{1}{2}M\left(\omega_s^2 + \omega_0^2\right)\left(\frac{1}{2}A_s^2 + \frac{1}{2}B_s^2\right)\right] \quad (5)$$

If ($\Gamma \approx 0$), the exponential decay term is approximately constant during an oscillation; in this case we have:

$$\left(q(t)_{damp}\right) \approx \left(\cos t\right)\left[A_s \sin(\omega_s t) + B_s \cos(\omega_s t)\right] \quad (6)$$

The energy (with $\omega_s \approx \omega_0$) is

$$E(t) \approx E_0 = \left[\frac{1}{2}M\left(2\omega_s^2\right)\left(\frac{1}{2}A_s^2 + \frac{1}{2}B_s^2\right)\right] = \frac{1}{2}M\omega_s^2\left(\frac{1}{2}A_s^2 + \frac{1}{2}B_s^2\right) \quad (7)$$

Note here again two energy components. A solution with two terms exists also in the driven oscillator (always with damping). Away from the initial phase, the oscillator is adapted to the external force, it oscillates with frequency $\omega_a = \omega_{est} \neq \omega_0$ and admits of a solution with two components: one absorptive (A_{abs}), and the other elastic (A_{el}). It is:

$$\left(q(t)_{adaptive}\right) = \left[A_{ass} \sin(\omega_a t) + B_{el} \cos(\omega_a t)\right] \quad (8)$$

with a mean energy (in a cycle) given by:

$$\left\langle E(t)\right\rangle_{T_a} = \left[\frac{1}{2}M\left(\omega_a^2 + \omega_0^2\right)\left(\frac{1}{2}A_{ass}^2 + \frac{1}{2}A_{el}^2\right)\right] \quad (9)$$

So energy is expressed through the two absorptive and elastic components.
If $\Gamma \approx 0$ and ($\omega_a = \omega_{est}$) $\approx \omega_0$, the oscillator has an adaptive oscillation (resonance) with two energy components:

$$\left\langle E(t)\right\rangle_{T_a} \approx \left[\frac{1}{2}M\left(\omega_a^2\right)\left(A_{ass}^2 + A_{el}^2\right)\right] \quad (10)$$

where it is $T_a = 2\pi/\omega_a$. Finally, we consider an oscillator belonging to a chain of elastically coupled oscillators . We think each oscillator of the chain as an oscillator driven by the neighboring oscillators, with imposed frequency $\omega_{est} \neq \omega_0$ (where ω_0 is the frequency of the single isolated oscillator). We'll say that the system of coupled oscillators got well adapted to the "external" frequency ($\omega_{est} = \omega_a$), where ω_a is now the frequency of one of the mode oscillations of the chain. It follows that the single filed-oscillator is like an oscillator (with damping coefficient $\Gamma \approx 0$) adapted to any frequency ω_a. In conclusion, the field-oscillator is an adapted oscillator which is described by two oscillating components. We return to the classical oscillator expressed in $(\mathbf{a}, \mathbf{a}^+)$ complex coordinates; it's:

$$a = \left[\left(\sqrt{\frac{m\omega}{2\hbar}}\right)\left(q + \frac{ip}{m\omega}\right)\right] \quad , \quad a^+ = \left[\left(\sqrt{\frac{m\omega}{2\hbar}}\right)\left(q - \frac{ip}{m\omega}\right)\right] \tag{11}$$

With the equations of motion:

$$\begin{cases} \left(\frac{da_t}{dt}\right) = -i\omega a_t \\ \left(\frac{da_t^+}{dt}\right) = i\omega a_t^+ \end{cases} \tag{12}$$

and with solutions

$$(a_t = a_0 \exp(-i\omega t) \quad , \quad a_t^+ = a_0^+ \exp(i\omega t)\ ) \tag{13}$$

where $[a_0 = a(0)\ ,\ a_0^+ = a^+(0)]$. In the coordinate plane (q, p), the solutions $(\mathbf{a_t}, \mathbf{a_t^+})$ could be related to a pair of vectors $(\mathbf{Oa}, \mathbf{Oa^+})$ with opposite phase rotation.

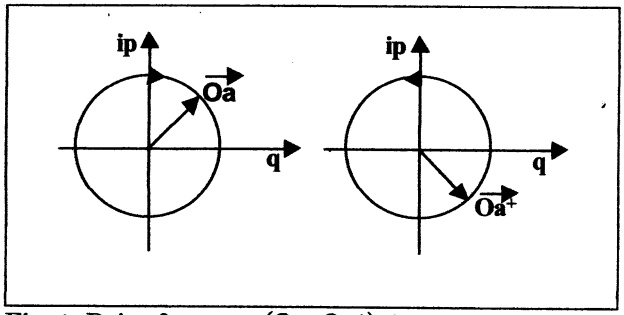


Fig. 1: Pair of vectors $(\mathbf{Oa}, \mathbf{Oa^+})$, in the coordinate plane (q, p).

The two solutions can be connected with the two opposite rotations of the phase.
In classical mechanics this two "rotations" aren't evident.

The two possibilities are hidden: an oscillating pendulum going back in time (clockwise rotation of the representative curve in Fig.1) is indistinguishable from the one going ahead in time (anticlockwise rotation). One feels that a new physics can be worked out, in which these two different possibilities (states) may be revealed simultaneously, as well as individually, in some physical phenomena. In this new "original physics" the two different possibilities would be expressed by some difference of the vectors **(Oa)** and **(Oa^+)** of the phase rotation:

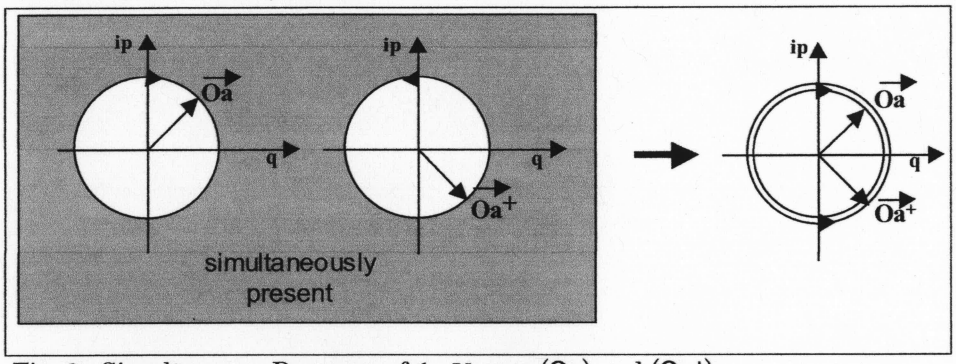


Fig. 2 : Simultaneous Presence of the Vectors **(Oa)** and **(Oa^+)**

The new physics is *quantum mechanics*, where the possible local states of a system are individually existing, but complementary at the experimental level, or else they can exist also in a superposition state (non-locality). About quantum oscillators, we think that [1] the **($Oa \neq Oa^+$)** difference is related to the commutation bracket (non-zero value) of the corresponding Hermitian operators **($\hat{a}$, $\hat{a}^+$)**:

$$\left[\hat{a}, \hat{a}^+\right] = 1 \tag{14}$$

We also feel that the two different directions of rotation, of the phase assigned to a quantum oscillator (note that the energy is degenerate in those two eigenstates), may instead individuate, under certain conditions, an observable event. To discover these special conditions we must review the quantum oscillator theory. The solutions [1] of the equations of the motion in the phase plane of the quantum Hermitian operators $(\hat{q}, \hat{p})$ are:

$$\left\{ \begin{array}{l} \hat{q}(t) = \hat{q}(0)\cos\omega t + \left(\dfrac{\hat{p}(0)}{m\omega}\right)\sin\omega t \\ \hat{p}(t) = \hat{p}(0)\cos\omega t - (m\omega)\hat{q}(0)\sin\omega t \end{array} \right\} \tag{15}$$

In an adaptive oscillator, the solutions (see eq. (8)) of the classical oscillator are:

$$q(t)_{adapt} = q_{el}(0)\cos(\omega t) + q_{abs}(0)\sin(\omega t) \tag{16}$$

For a quantum oscillator, expressed by operators, it will be:

$$\hat{q}(t) = \hat{q}_{el}(0)\cos(\omega t) + \hat{q}_{abs}(0)\sin(\omega\, t) \tag{17}$$

where q_{el} is the elastic component in-phase with the external force, while q_{abs} has a phase delay of $\pi/2$ with respect to (w.r.t.) the external force. We remember that any oscillator has two components: elastic and inertial; therefore we admit the following equivalence: $q_{abs} \equiv q_{inertial}$. It follows

$$\hat{q}(t) = \hat{q}_{el}(0)\cos(\omega t) + \hat{q}_{in}(0)\sin(\omega t) \tag{18}$$

This equation can also describe, as we have already said, an oscillator belonging to a chain of coupled oscillators (field). We say that a **quantum field-oscillator** should always be represented with two components. The graphic representation of the quantum oscillator with two components is:

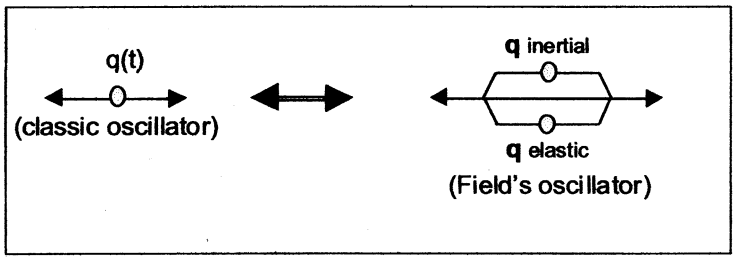


Fig. 3: Quantum oscillator with two components

Considering (13), using (11) with operators, and resorting to the Euler's relations, easily one finds that:

$$\begin{cases} \hat{a}_t = \left(\sqrt{\dfrac{m\omega}{2\hbar}}\right)\left[\hat{q}_0 e^{-i\omega t} + i\left(\dfrac{\hat{p}_0}{m\omega}\right)e^{-i\omega t}\right] = \hat{k}_q e^{-i\omega t} + i\hat{k}_p e^{-i\omega t} = \hat{k}_q e^{-i\omega t} + \hat{k}_p e^{-i(\omega t - \pi/2)} \\ \hat{a}_t^{+} = \left(\sqrt{\dfrac{m\omega}{2\hbar}}\right)\left[\hat{q}_0 e^{i\omega t} - i\left(\dfrac{\hat{p}_0}{m\omega}\right)e^{i\omega t}\right] = \hat{k}_q e^{i\omega t} - i\hat{k}_p e^{i\omega t} = \hat{k}_q e^{i\omega t} + \hat{k}_p e^{i(\omega t - \pi/2)} \end{cases} \tag{19}$$

where it's [q_0= q(0), p_0= p(0)] and

$$\left\{ \begin{array}{c} \left(\hat{k}_q = \left(\sqrt{\dfrac{m\omega}{2\hbar}}\right)\hat{q}_0\,,\ \hat{k}_p = \left(\sqrt{\dfrac{1}{2m\hbar\omega}}\right)\hat{p}_0\right) \\ \left[\hat{k}_q\hat{k}_p - \hat{k}_p\hat{k}_q\right] = \dfrac{i}{2} \\ \left[\hat{a}_0, \hat{a}_0^{+}\right] = 1 \end{array} \right\} \tag{20}$$

Thus, we notice that the $(\hat{a}, \hat{a}^+)$ operators can be expressed by two components (see also (15) and (18)): an elastic component and an inertial one.
We could say then that the operators $(\hat{a}, \hat{a}^+)$ will be expressed by the following equation:

$$\begin{cases} \hat{a}_t = \hat{k}_{(elastic)} e^{-i\omega t} + \hat{k}_{(inertial)} e^{-i(\omega t - \pi/2)} \\ \hat{a}_t^+ = \hat{k}_{(elastic)} e^{i\omega t} + \hat{k}_{(inertial)} e^{i(\omega t - \pi/2)} \end{cases} \tag{21}$$

It is easy to show (taking into account the present relations into (20) that $[\hat{a}_t, \hat{a}_t^+] = 1$.
We could then say that the operators $(\hat{a}, \hat{a}^+)$ will be expressed by the following equation (see (21)):

$$\left\{ \begin{array}{l} \hat{a}_t = \hat{a}(t)_{elastic} + \hat{a}(t)_{inertial} \equiv O\vec{a} \\ \hat{a}_t^+ = \hat{a}^+(t)_{elastic} + \hat{a}^+(t)_{inertial} \equiv O\vec{a}^+ \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \hat{a}_t = \hat{a}_{(el)} e^{-i\omega t} + \hat{a}_{(in)} e^{-i(\omega t - \pi/2)} \\ \hat{a}_t^+ = \hat{a}^+_{(el)} e^{i\omega t} + \hat{a}^+_{(in)} e^{i(\omega t - \pi/2)} \end{array} \right\} \tag{22}$$

This is the generalization of (21). Let's duly observe that the elastic component has a phase delay of $\pi/2$, in comparison to the inertial one. Hereafter, we will resort to an oscillator with two components to describe an field-oscillator associated with a particle.
We conjecture that the oscillator field, with two components, could allow us to detect the phase rotation during the interactions between particles.

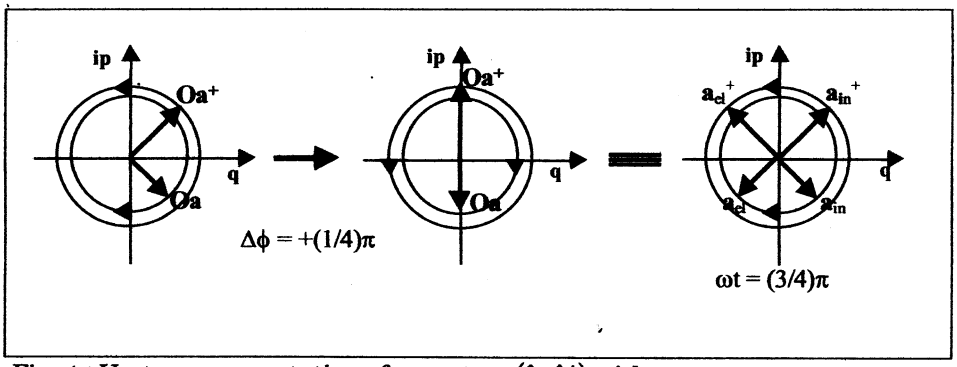


Fig. 4 : Vector representation of operators $(\hat{a}, \hat{a}^+)$ with two components,

We think that a quantum oscillator has two oscillating components also in the vacuum state.
Therefore, if the vacuum state is with energy $\varepsilon = 1/2\ \hbar\omega$, to each component will go the energy $\varepsilon = 1/4\ \hbar\omega$. Let us define **"full semi-quantum (•)"** the quantum of oscillation with energy $\varepsilon = 1/2\ \hbar\omega$, and **"empty semi-quantum (o)"** the quantum with energy $\varepsilon = 1/4\ \hbar\omega$.
Remember that the $(\hat{a})$ operator is an annihilation operator while $(\hat{a}^+)$ is a creation operator with the Hamiltonian operator:

$$\hat{H} = \left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right)\hbar\omega = \left(\hat{n} + \frac{1}{2}\right)\hbar\omega \tag{23}$$

where **(n)** is the operator number of the quanta. The operators **(H)** and **(n)**, endowed with simultaneously definable eigenvectors, are commuting. In fact the eigenvalues of the Hamiltonian operator are:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \tag{24}$$

where n (1,2,...) are the eigenvalues of the **(n)** operator.
As a novelty, in any type of oscillation, we introduce an operator **(r)** expressing the direction of the phase rotation: this operator has the eigenvalues r' = ±1 and commutes with **(H)** and, accordingly, with **(n)**. In a quantum isolated oscillator, the energy can degenerate into eigenvalues of **(r)**, as it is degenerated in the direction of the phase rotation. We shall have [2]:

$$\left[E_n = \left(n + \frac{1}{2}\right)\hbar\omega\right]_{(r'=\pm 1)} \tag{25}$$

Therefore we define a $\mathbf{S(H,n)}_{(r'=\pm 1)}$ vectorial space, with as a base the eigenvectors of the $\mathbf{(H, n)}_{(r'=\pm 1)}$ operators, where is

$$[(|\Psi>)_{(r'=\pm 1)} \equiv (|\Psi_H>|\Psi_n>)_{(r'=\pm 1)}\ ]$$

Let us recall that we name **"IQO"** the field oscillator defined in terms of semi-quanta [where "IQO" is the acronym of "**I**ntrinsec **Q**uantum **O**scillator"].

2 The Intrinsec Quantum Oscillator (IQO)

Dirac ([2], pp 156-157) proposed the following relations for the **(a, a⁺)** operators:

$$\begin{cases} \hat{a} = \left(\exp(i\hat{\varphi})\right)\left(\sqrt{\hat{n}}\right) \\ \hat{a}^{+} = \left(\sqrt{\hat{n}}\right)\left(\exp(-i\hat{\varphi})\right) \end{cases} \tag{26}$$

where the **φ**-operator of the phase angle has been introduced. However, it's evident that this operator **(φ)** does not commute with the operators **(H)**, **(n)** and **(r)**: the phase angle [θ = ωt] is uncertain, if, instead, the (n)-eigenvalue of **(n)** is certain. Notice that, if the energy of the oscillator (degenerated into an eigenvalue of **(r)**) is certain, then the phase is uncertain.
However, the (26) (see Glover [2]) leads to some contradictions: this seems to push us to regard the phase as a physical variable and not operator.

In [2] it is defined the functional operator (**Φ**)

$$\hat{\Phi}(\varphi) = \exp(i\varphi) \quad , \quad \hat{\Phi}^{+}(\varphi) = \exp(-i\varphi) \tag{27}$$

Now we have

$$\begin{cases} \hat{a} = (\exp(i\varphi))(\sqrt{\hat{n}}) \\ \hat{a}^{+} = (\sqrt{\hat{n}})(\exp(-i\varphi)) \end{cases} \tag{28}$$

where the variables (n) and (φ) are related by an uncertainty relation. However (see the work of Fain in bibl. [2]), the energy is once more degenerated into the sign of (φ) and it's not possible to determine the direction of phase rotation. Instead, we think that the IQO- field allows us to detect the direction of the phase rotation thanks to his "double" structure (remember the two components). To this aim, a new form of the operator (**Φ**) is introduced because in it the operator (**r**), besides the variable phase (φ), is defined. We propose the form:

$$\begin{cases} \hat{a} = (\hat{\Phi}(\hat{r},\varphi))(\sqrt{\hat{n}}) \\ \hat{a}^{+} = (\sqrt{\hat{n}})(\hat{\Phi}^{+}(\hat{r},\varphi)) \end{cases} \tag{29}$$

where the Φ-operator function consists in

$$\hat{\Phi}(\hat{r},\varphi) = \exp(i\hat{r}\varphi) \quad , \quad \hat{\Phi}^{+}(\hat{r},\varphi) = \exp(-i\hat{r}\varphi) \tag{30}$$

In a IQO, it follows (see (21)):

$$\begin{cases} \hat{a} = (\Phi(\hat{r},\varphi))(\sqrt{\hat{n}}) = \left[\left(e^{-i\hat{r}\omega t}\right)(\hat{k}_{el} + i\hat{k}_{in})\right] \\ \hat{a}^{+} = (\sqrt{\hat{n}})\Phi^{*}(\hat{r},\varphi) = \left[(\hat{k}_{el} - i\hat{k}_{in})\left(e^{i\hat{r}\omega t}\right)\right] \end{cases} \tag{31}$$

But

$$\begin{cases} \hat{a}^{+}\hat{a} = \hat{n} = (\sqrt{\hat{n}})(\sqrt{\hat{n}}) \\ \hat{a}^{+}\hat{a} = \hat{n} = \left[(\hat{k}_{el} - i\hat{k}_{in})\left(e^{i\hat{r}\omega t}\right)\right] \cdot \left[\left(e^{-i\hat{r}\omega t}\right)(\hat{k}_{el} + i\hat{k}_{in})\right] = (\hat{k}_{el} - i\hat{k}_{in})(\hat{k}_{el} + i\hat{k}_{in}) \end{cases} \tag{32}$$

Note that the last equality implies a contradiction with the first one, because it's $(k_{el}\text{-}ik_{in}) \neq (k_{el}+ik_{in})$. To avoid this contradiction, we recall what has been assumed previously: the difference (**Oa ≠ Oa⁺**) between vectors (**Oa , Oa⁺**) is contained implicitly in the commutator [**â, â⁺**] (non-zero value). Because the no commutation [**â, â⁺**] is a physical reality, then the difference (**Oa ≠ Oa⁺**) should be admitted. This means that we can distinguish the norm of these operators or we can distinguish the $(\sqrt{\hat{n}})$ operator contained in (**â**) from that contained in (**â⁺**).

Then we can now conjecture the following form:

$$\left\{\begin{array}{c} \hat{a}=\hat{\Phi}(\hat{r},\varphi)\left(\sqrt{\hat{n}_{+}}\right) \\ \hat{a}^{+}=\left(\sqrt{\hat{n}_{-}}\right)\left(\hat{\Phi}^{+}(\hat{r},\varphi)\right) \end{array}\right\} \tag{33}$$

where

$$\sqrt{\hat{n}_{+}}=(\hat{k}_{el}+i\hat{k}_{in})\ \ ;\sqrt{\hat{n}_{-}}=(\hat{k}_{el}-i\hat{k}_{in}) \tag{34}$$

with

$$\left\{\begin{array}{c} \sqrt{\hat{n}_{+}}\neq\sqrt{\hat{n}_{-}} \\ \left(\sqrt{\hat{n}_{+}}\right)^{+}=\sqrt{\hat{n}_{-}}\Leftrightarrow\left(\sqrt{\hat{n}_{-}}\right)^{+}=\sqrt{\hat{n}_{+}} \end{array}\right\} \tag{35}$$

We'll have

$$\left(\sqrt{\hat{n}_{+}}\right)\left(\sqrt{\hat{n}_{-}}\right)=(\hat{k}_{el}+i\hat{k}_{in})\ (\hat{k}_{el}-i\hat{k}_{in})=\left(\hat{k}_{el}^{2}+\hat{k}_{in}^{2}\right)-\left(\frac{1}{2}\right) \tag{36}$$

Thus, we find (by eq. (31) and (33)) that

$$\left\{\begin{array}{l} \hat{a}^{+}\hat{a}=\left[\left(\hat{k}_{el}\right)\left(e^{i\hat{r}\omega t}\right)-\left(i\hat{k}_{in}\right)\left(e^{i\hat{r}\omega t}\right)\right]\left[\left(e^{-i\hat{r}\omega t}\right)\hat{k}_{el}+\left(ie^{-i\hat{r}\omega t}\right)\hat{k}_{in}\right]=\left[\left(\sqrt{\hat{n}_{+}}\right)\left(\sqrt{\hat{n}_{-}}\right)\right] \\ \hat{a}^{+}\hat{a}=\hat{n} \end{array}\right. \tag{37}$$

confirming that

$$\hat{n}=\left[\left(\sqrt{\hat{n}_{+}}\right)\left(\sqrt{\hat{n}_{-}}\right)\right] \tag{38}$$

In this way, eq. (33) becomes:

$$\left\{\begin{array}{l} \hat{a}=(\hat{\Phi}(\hat{r},\varphi))(\sqrt{\hat{n}_{+}})=\left[\left(e^{-i\hat{r}\omega t}\right)(\hat{k}_{el}+i\hat{k}_{in})\right] \\ \hat{a}^{+}=(\sqrt{\hat{n}_{-}})\hat{\Phi}^{+}(\hat{r},\varphi)=\left[(\hat{k}_{el}-i\hat{k}_{in})\left(e^{i\hat{r}\omega t}\right)\right] \end{array}\right. \tag{39}$$

A new Dirac's form of the (**a**, **a**$^{+}$) operators will be

$$\left\{\begin{array}{c} \hat{a}=(\hat{\Phi}(\hat{r},\varphi)\left(\sqrt{\hat{n}_{+}}\right)=\left[\left(\hat{\Phi}(\hat{r},\varphi)\right)\left(\hat{k}_{el}\right)+\left(\hat{\Phi}(\hat{r},\varphi)\right)\left(i\hat{k}_{in}\right)\right]=\left(\hat{a}_{el}+\hat{a}_{in}\right) \\ \hat{a}^{+}=\left(\sqrt{\hat{n}_{-}}\right)\left(\hat{\Phi}^{+}(\hat{r},\varphi)\right)=\left[\left(\hat{k}_{el}\right)\left(\hat{\Phi}^{+}(\hat{r},\varphi)\right)-\left(i\hat{k}_{in}\right)\left(\hat{\Phi}^{+}(\hat{r},\varphi)\right)\right]=\left(a_{el}^{+}+\hat{a}_{in}^{+}\right) \end{array}\right. \tag{40}$$

Note that the (**a**, **a**$^{+}$) becomes complex operators, composed by the operator (Φ) (with real norm $\rho = 1$) and by the $\left(\sqrt{\hat{n}_{\pm}}\right)$ operator.

This form is also consistent with the actions of the operators (**a**, $\mathbf{a^+}$) on the eigenvectors $|\mathbf{n}>$ of the operator (**n**):

$$\left\{\begin{array}{l} \hat{a}|n\rangle=\left(\sqrt{n}\right)|n-1\rangle \\ \hat{a}^{+}|n\rangle=\left(\sqrt{n+1}\right)|n+1\rangle \end{array}\right\} \tag{41}$$

Notice that this new form of the operators (**a**, $\mathbf{a^+}$) contains an information more (see the $\left(\sqrt{\hat{n}_{\pm}}\right)$) than the old form. **This shows that the field-oscillator with two components has an internal space that can be detected by some phenomenon.** Note that in an eigenstate of the operators (**H, n, r**), is possible to describe oscillators with their semi-quanta (one elastic and other inertial), and the empty ones! Always in the eigenstate of (**H, n, r**) it can be easily demonstrated [1], by the properties of the quantum operators, that

$$\left\{\begin{array}{l} \hat{U}|n\rangle=\left(\frac{1}{2}m\omega^2\hat{x}^2\right)|n\rangle=\left(\frac{1}{4}\hbar\omega\right)_U\left[\hat{a}^2+(\hat{a}^{+})^2+\hat{a}^{+}\hat{a}+\hat{a}\,\hat{a}^{+}\right]|n\rangle \\ \hat{K}|n\rangle=\left(\frac{\hat{p}^2}{2m}\right)|n\rangle=\left(\frac{1}{4}\hbar\omega\right)_K\left[-\hat{a}^2-(\hat{a}^{+})^2+\hat{a}^{+}\hat{a}+\hat{a}\,\hat{a}^{+}\right]|n\rangle \end{array}\right\} \tag{42}$$

So we can calculate the value of the potential and kinetics energy in the $|0>$ eigenstate:

$$\left\{\begin{array}{l} \hat{U}|0\rangle=\left(\frac{1}{2}m\omega^2\hat{x}^2\right)|0\rangle=\left(\frac{1}{4}\hbar\omega\right)_U\left[(\hat{a}^{+})^2+1\right]|0\rangle \\ \hat{K}|0\rangle=\left(\frac{\hat{p}^2}{2m}\right)|0\rangle=\left(\frac{1}{4}\hbar\omega\right)_K\left[-(\hat{a}^{+})^2+1\right]|0\rangle \end{array}\right\} \tag{43}$$

Thus the form of the Hamiltonian operator is found:

$$\hat{H}|0\rangle=\hat{U}|0\rangle+\hat{K}|0\rangle=\left(\frac{1}{4}\hbar\omega\right)_U|0\rangle+\left(\frac{1}{4}\hbar\omega\right)_K|0\rangle \tag{44}$$

By contrast, in the $|n>$ eigenstate it is

$$\hat{U}|n\rangle+\hat{K}|n\rangle=\left(\frac{1}{4}\hbar\omega\right)_U\left[\hat{a}^{+}\hat{a}+\hat{a}\,\hat{a}^{+}\right]_U|n\rangle+\left(\frac{1}{4}\hbar\omega\right)_K\left[\hat{a}^{+}\hat{a}+\hat{a}\,\hat{a}^{+}\right]_K|n\rangle \tag{45}$$

We demonstrate also that

$$\hat{H}|n\rangle=\hat{U}|n\rangle+\hat{K}|n\rangle=\left(2\hat{n}+\hat{1}\right)\left(\frac{1}{4}\hbar\omega\right)_U|n\rangle+\left(2\hat{n}+\hat{1}\right)\left(\frac{1}{4}\hbar\omega\right)_K|n\rangle \tag{46}$$

with eigenvalues in the eigenstate S(**H**,**n**) such that

$$\left[H_{(n)}\right]_{r'} = \left[U_{(n)} + K_{(n)}\right]_{r'} = \left[(2n+1)\left(\frac{1}{4}\hbar\omega\right)_U + (2n+1)\left(\frac{1}{4}\hbar\omega\right)_K\right]_{r'} \tag{47}$$

$$\left(H_{(n)}\right)_{r'} = \left(U_{(n)} + K_{(n)}\right)_{r'} = \left[n\left(\left(\frac{1}{2}\hbar\omega\right)_U\right) + \left(\frac{1}{4}\hbar\omega\right)_U\right]_{r'} + \left[n\left(\left(\frac{1}{2}\hbar\omega\right)_K\right) + \left(\frac{1}{4}\hbar\omega\right)_K\right]_{r'} \tag{48}$$

Here it is a further confirmation that in the quantum oscillator there are some full semi-quanta and some empty semi-quanta.
Besides [3], looking at the forms of the wave functions $\Psi(x)$ and $|\Psi_1(x)|^2$ of the quantum oscillator in the $|1\rangle$ eigenstate, we note that the probability function $P_1(x) = |\Psi_1(x)|^2$ can be interpreted (notice the central node) as a function describing an oscillator with two basic units of oscillations (**sub-oscillators**).

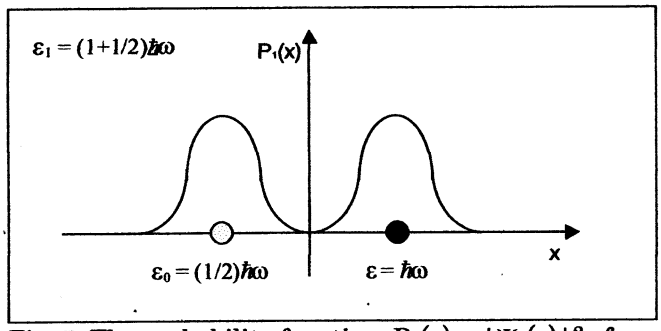


Fig. 5: The probability function $P_1(x) = |\Psi_1(x)|^2$ of quantum oscillator

The [$\varepsilon_1 = (1)\hbar\omega + (1/2)\hbar\omega$] energy is distribute in two sub-oscillators, of which one (sub-quantum) with energy [$\varepsilon = (1/2)\hbar\omega$], and the other with energy [$\varepsilon = (1)\hbar\omega$]. In the $|0\rangle$ eigenstate [3] there is only one sub-oscillator:

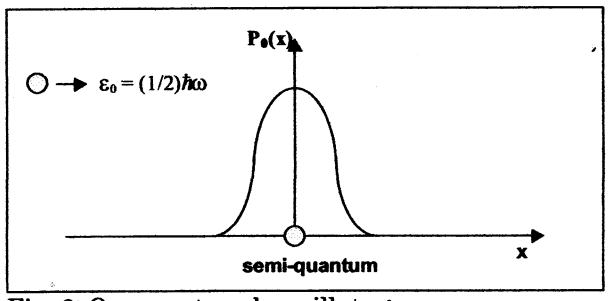


Fig. 6: One empty sub-oscillator:

Looking at the eq. (46) and (47), one gets

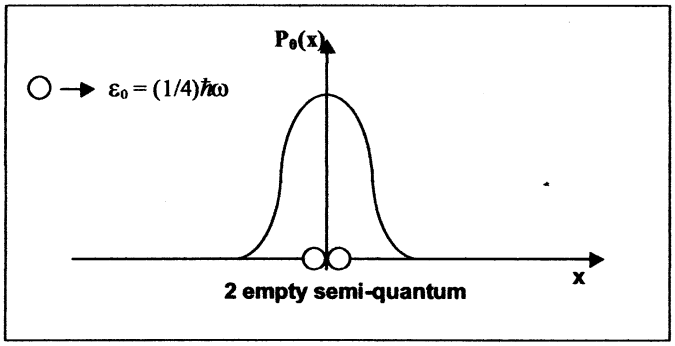


Fig. 7: A sub- oscillator

As

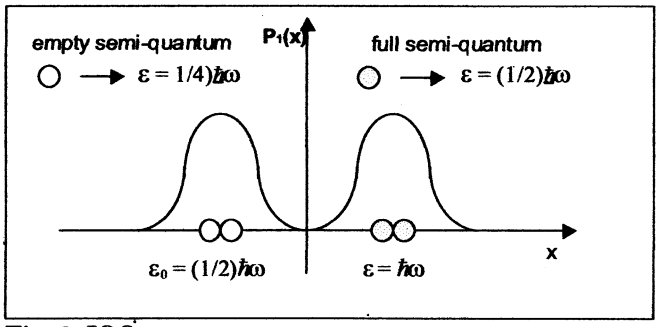


Fig. 8: IQO

In the $|n>$ state the number of coupled oscillators is (n+1), where n is the number of quanta.

In summary, we can affirm that an oscillator going to the n-level will consists of (n +1) sub-oscillators: one of them has two (**o**) empty semi-quanta with energy $\varepsilon = 1/4\ \hbar\omega$, while the n remaining ones have each two full semi-quanta (•) with energy $\varepsilon = 1/2\ \hbar\omega$ (note that two full semi-quanta are equal to one quantum or [(•) + (•) = 1 quantum]).

Even if the maximum potential energy, and the maximum kinetic energy, always have a constant value over time, the semi-quantum goes back and forth from a sub-oscillator to another. Now we can build the following correspondences:

$$\left\{\varepsilon_{|1>} = \begin{matrix} U \equiv (\bullet + o) \\ K \equiv (\bullet + o) \end{matrix}\right\} <=> \left\{\vec{\Psi}_{osc} \equiv \begin{pmatrix} \vec{O}a = \vec{O}a_{el} + \vec{O}a_{in} \\ \vec{O}a^{+} = \vec{O}a_{el}^{+} + \vec{O}a_{in}^{+} \end{pmatrix}\right\} <=> \left\{\begin{pmatrix} a_{el} \equiv U_{el}(\bullet) & a_{in} \equiv K_{in}(o) \\ a_{el}^{+} \equiv U_{el}(o) & a_{in}^{+} \equiv K_{in}(\bullet) \end{pmatrix}\right\} \quad (49)$$

There is also the following equivalence (invariance for (o) ←→ (•) change) between matrices:

$$\left\{\begin{pmatrix} a_{el} \equiv U_{el}(\bullet) & a_{in} \equiv K_{in}(\mathrm{o}) \\ a_{el}^{+} \equiv U_{el}(\mathrm{o}) & a_{in}^{+} \equiv K_{in}(\bullet) \end{pmatrix}\right\} <\equiv> \left\{\begin{pmatrix} a_{el} \equiv U_{el}(\mathrm{o}) & a_{in} \equiv K_{in}(\bullet) \\ a_{el}^{+} \equiv U_{el}(\bullet) & a_{in}^{+} \equiv K_{in}(\mathrm{o}) \end{pmatrix}\right\} \tag{50}$$

Looking at fig. 4, it holds the following (vectorial) representation

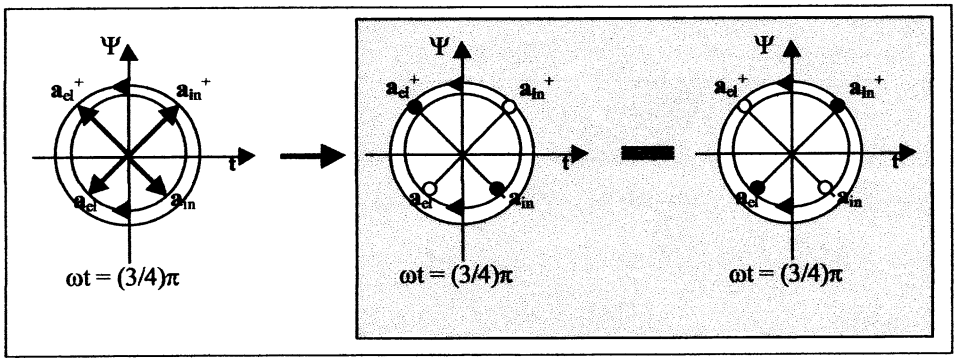


Fig. 9

About the $(\mathbf{a}, \mathbf{a}^{+})$ operators we can set the following symbols:

$$\underset{B_1\text{ - Matrix}}{\left\{\begin{pmatrix} \hat{a}_{el} \equiv (\hat{\bullet})_{el} & \hat{a}_{in} \equiv (\hat{\mathrm{o}})_{in} \\ \hat{a}_{el}^{+} \equiv (\hat{\mathrm{o}}^{+})_{el} & \hat{a}_{in}^{+} \equiv (\hat{\bullet}^{+})_{in} \end{pmatrix}\right\}} <\equiv> \underset{B_2\text{ - Matrix}}{\left\{\begin{pmatrix} \hat{a}_{el} \equiv (\hat{\mathrm{o}})_{el} & \hat{a}_{in} \equiv (\hat{\bullet})_{in} \\ \hat{a}_{el}^{+} \equiv (\hat{\bullet}^{+})_{el} & \hat{a}_{in}^{+} \equiv (\hat{\mathrm{o}}^{+})_{in} \end{pmatrix}\right\}} \tag{51}$$

where $((\mathbf{o}^{+})_{el}, (\mathbf{o}^{+})_{in})$ are the creation operators of empty semi-quanta (with $\varepsilon = 1/4\ \hbar\omega$), while $((\bullet^{+})_{el}, (\bullet^{+})_{in})$ are the creation operators of full semi-quanta ($\varepsilon = 1/2\ \hbar\omega$). Instead, $((\mathbf{o})_{el}, (\mathbf{o})_{in})$ are annihilation operator of empty semi-quanta, while $((\bullet)_{el}, (\bullet)_{in})$ are the annihilation operator of full semi-quanta. For an eigenstate of $(\mathbf{r})$ and $(\mathbf{n})$ it will be (see (21) and (22))

$$\begin{cases} \overrightarrow{\mathrm{O}\hat{a}}_{n,r'}^{+}(t) \equiv \hat{a}_{n,r'}^{+}(t) = (\hat{\bullet}^{+})_{el}^{(n)}\left[\exp(ir'\omega t)\right] + (\hat{\mathrm{o}}^{+})_{in}^{(n)}\left[\exp(i(r'\omega t - \pi/2))\right] \\ \overrightarrow{\mathrm{O}\hat{a}}_{n,r'}(t) \equiv \hat{a}_{n,r'}(t) = (\hat{\mathrm{o}})_{el}^{(n)}\left[\exp(-ir'\omega t)\right] + (\hat{\bullet})_{in}^{(n)}\left[\exp(-i(r'\omega t - \pi/2))\right] \end{cases} \tag{52}$$

or equivalently

$$\begin{cases} \overrightarrow{\mathrm{O}\hat{a}}_{n,r'}^{+}(t) \equiv \hat{a}_{n,r'}^{+}(t) = (\hat{\mathrm{o}}^{+})_{el}^{(n)}\left[\exp(ir'\omega t)\right] + (\hat{\bullet}^{+})_{in}^{(n)}\left[\exp(i(r'\omega t - \pi/2))\right] \\ \overrightarrow{\mathrm{O}\hat{a}}_{n,r'}(t) \equiv \hat{a}_{n,r'}(t) = (\hat{\bullet})_{el}^{(n)}\left[\exp(-ir'\omega t)\right] + (\hat{\mathrm{o}})_{in}^{(n)}\left[\exp(-i(r'\omega t - \pi/2))\right] \end{cases} \tag{53}$$

These mathematical relations give to us the mathematical form of the field oscillator called **"IQO"**.

For an IQO it will be

$$\begin{cases} \vec{\Psi}_{IQuo}(t)_{n,r'} = \vec{O}\hat{a}_{n,r'}(t) + \vec{O}\hat{a}^{+}_{n,r'}(t) & \text{eigenstate} \mid n,r'> \\ \vec{\Psi}_{IQuo}(t) = \vec{O}\hat{a}(t) + \vec{O}\hat{a}^{+}(t) & \text{superposition state} \end{cases} \tag{54}$$

In an IQO, at the first level it is

$$\begin{cases} \hat{a}^{+}_{1,r'}(t) = (\hat{\bullet}^{+})_{el}\left[\exp(ir'\omega t)\right] + (\hat{o}^{+})_{in}\left[\exp(i(r'\omega t - \pi/2))\right] \\ \hat{a}_{1,r'}(t) = (\hat{o})_{el}\left[\exp(-ir'\omega t)\right] + (\hat{\bullet})_{in}\left[\exp(-i(r'\omega t - \pi/2))\right] \end{cases} \tag{55}$$

We'll have

$$(\varepsilon_1)_{r'} = \begin{Bmatrix} U \equiv (\bullet + o) \\ K \equiv (\bullet + o) \end{Bmatrix}_{r'} <=> \begin{Bmatrix} \hat{a}_{el} \equiv U_{el}(\hat{o}) & \hat{a}_{in} \equiv K_{in}(\hat{\bullet}) \\ a^{+}{}_{el} \equiv U_{el}(\hat{\bullet}) & a^{+}{}_{in} \equiv K_{in}(\hat{o}) \end{Bmatrix}_{r'} \tag{56}$$

$$\begin{aligned} &(\hat{r})' = +1 => \left[\begin{matrix} [\hat{a}_{el} = \hat{o}_{el}(\text{clockwise direction})] \\ [\hat{a}_{in} = \hat{\bullet}_{in}(\text{clockwise direction})] \end{matrix}\right] \\ &(\hat{r})' = +1 => \left[\begin{matrix} [\hat{a}^{+}{}_{el} = \hat{\bullet}^{+}{}_{el}(\text{anti-clockwise direction})] \\ [\hat{a}^{+}{}_{in} = \hat{o}^{+}{}_{in}(\text{anti-clockwise direction})] \end{matrix}\right] \end{aligned} \tag{57}$$

$$\begin{aligned} &(\hat{r})' = -1 => \left[\begin{matrix} [\hat{a}_{el} = \hat{o}_{el}(\text{anti-clockwise direction})] \\ [\hat{a}_{in} = \hat{\bullet}_{in}(\text{anti-clockwise direction})] \end{matrix}\right] \\ &(\hat{r})' = -1 => \left[\begin{matrix} [\hat{a}^{+}{}_{el} = \hat{\bullet}^{+}{}_{el}(\text{clockwise direction})] \\ [\hat{a}^{+}{}_{in} = \hat{o}^{+}{}_{in}(\text{clockwise direction})] \end{matrix}\right] \end{aligned} \tag{58}$$

We could use the following representation (see (58)) about an IQO with (r' = -1):

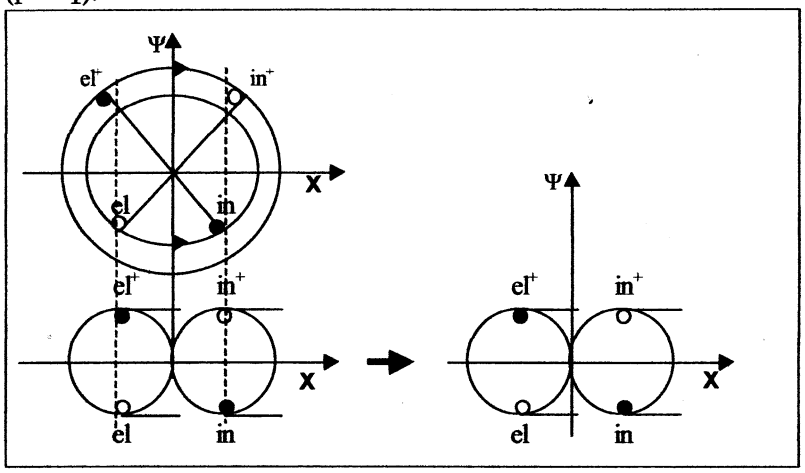


Fig. 10

This representation will be useful to represent a particle in motion along the x axis, seeing this like a quantum in motion along a line of coupled field-oscillators. Looking Fig. 3, Fig. 5, Fig. 6, Fig. 7, Fig. 8 and Fig. 10, we set forth the follows representation (representation with semi-quanta):

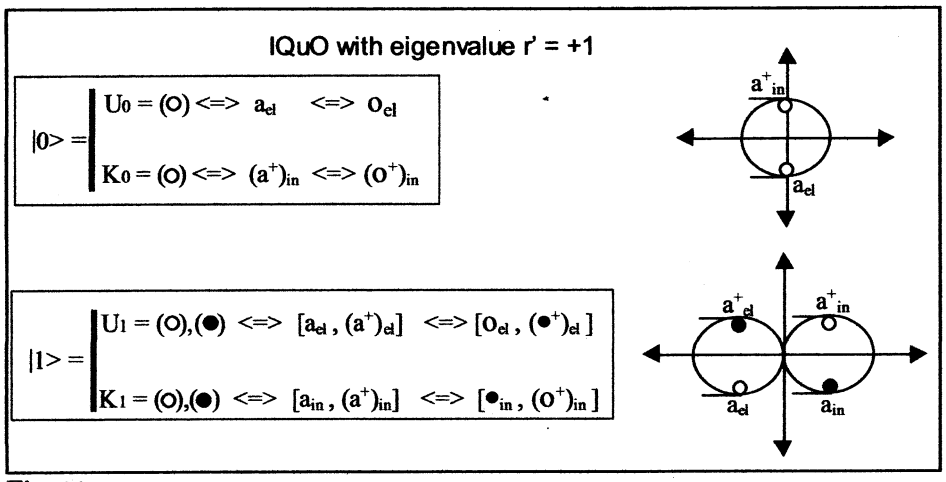


Fig. 11

The oscillator with two components (forced oscillator) establishes the presence of an internal degree of freedom.
Two different IQO in rotation directions r' = (+1, -1), are associated with two (r'). It's self-evident that an IQO couldn't change spontaneously the r' eigenvalue, passing from r' = +1 to r' = - 1.
Let's show (for example) the first three possible configurations of the IQO during the phase variation, with (r' = 1):

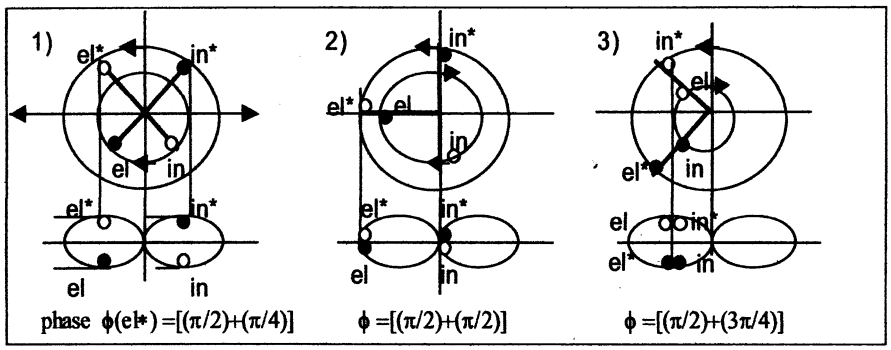


Fig. 12

In analogy with the classical oscillator, we project on the x axis the motion of the quanta which represent the oscillator; we'll have:

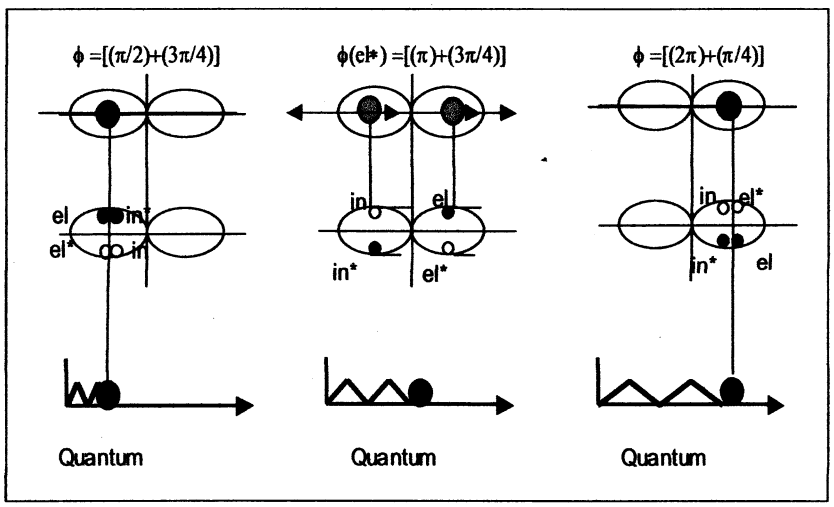


Fig. 13

where the quantum position follows the probabilistic law of function $|\Psi_1(x)|^2$. We underline that in the $|r>$ eigenstate (local state) the phase is uncertain (no local state of the possible configurations). The non-local state of the phase involves that the transition instant of a semi-quantum, from a sub-oscillation to other one, is uncertain, even if the configurations during the oscillation follow each other in an orderly way. Let us show the IQO's form with [r'] = - 1 eigenvalue:

$$\begin{cases} \hat{a} = e^{-i\vec{r}\omega t}\hat{k}_{el} + ie^{-i\vec{r}\omega t}\hat{k}_{in}\Big|_{r'=-1} = (\hat{k}_{el} + i\hat{k}_{in})e^{i\omega t} = (\hat{k}_{el} + i\hat{k}_{in}e^{i\pi/2})e^{i\omega t} \\ \hat{a}^{+} = \hat{k}_{el}e^{i\vec{r}\omega t} - i\hat{k}_{in}e^{i\vec{r}\omega t}\Big|_{r'=-1} = (\hat{k}_{el} - i\hat{k}_{in})e^{-i\omega t} = (\hat{k}_{el} + \hat{k}_{in}e^{-i\pi/2})e^{-i\omega t} \end{cases} \tag{59}$$

Now we show the IQO's configurations for:

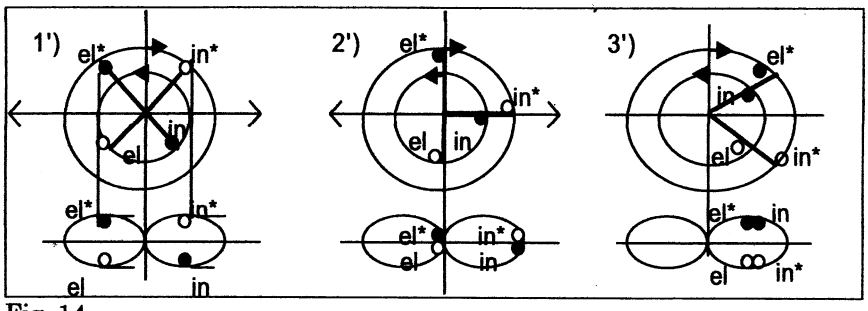


Fig. 14

We define the Φ(B)-matrix associated with each configuration:

$$\left[\Phi(B)\right]_{(r'=+1)} \equiv \left\{ \begin{pmatrix} (a_{el})_{(cl)} & (a_{in})_{(cl)} \\ (a^{+}_{el})_{(c\bar{l})} & (a^{+}_{in})_{(c\bar{l})} \end{pmatrix} <=> \left[\begin{pmatrix} (o_{el})_{(cl)} & (\bullet_{in})_{(cl)} \\ (\bullet^{+}_{el})_{(c\bar{l})} & (o^{+}_{in})_{(c\bar{l})} \end{pmatrix} \equiv \begin{pmatrix} (\bullet_{el})_{(cl)} & (o_{in})_{(cl)} \\ (o^{+}_{el})_{(c\bar{l})} & (\bullet^{+}_{in})_{(c\bar{l})} \end{pmatrix} \right] \right\} \tag{60}$$

$$\left[\Phi(B)\right]_{(r'=-1)} \equiv \left\{ \begin{pmatrix} (a_{el})_{(c\bar{l})} & (a_{in})_{(c\bar{l})} \\ (a^{+}_{el})_{(cl)} & (a^{+}_{in})_{(cl)} \end{pmatrix} <=> \left[\begin{pmatrix} (o_{el})_{(c\bar{l})} & (\bullet_{in})_{(c\bar{l})} \\ (\bullet^{+}_{el})_{(cl)} & (o^{+}_{in})_{(cl)} \end{pmatrix} \equiv \begin{pmatrix} (\bullet_{el})_{(c\bar{l})} & (o_{in})_{(c\bar{l})} \\ (o^{+}_{el})_{(cl)} & (\bullet^{+}_{in})_{(cl)} \end{pmatrix} \right] \right\} \tag{61}$$

where the cl index means clockwise direction, and the $\overline{cl}$ index means anti-clockwise direction .
Using the properties of the **(a, a+)** quantum operators (i.e. their commutators) and the properties of the **(H)** Hamiltonian operator, we obtain the commutation brackets of the **(•, o)** semi-quantum operators, as:

$$\begin{aligned} &\text{I° case} : \left[(\bullet^{+}_{el})_{cl}, (o_{el})_{\overline{cl}}\right] = 0 \quad ; \quad \left[(\bullet^{+}_{in})_{cl}, (o_{in})_{\overline{cl}}\right] = 0 \\ &\text{I° case} : \left[(o^{+}_{el})_{cl}, (\bullet_{el})_{\overline{cl}}\right] = 0 \quad ; \quad \left[(o^{+}_{in})_{cl}, (\bullet_{in})_{\overline{cl}}\right] = 0 \end{aligned} \tag{62}$$

The other relations about the semi-quantum operators and about their symmetry properties can be synthesized by the following commutation brackets:

$$\begin{aligned} &\text{I°) case} : \left[\bullet^{+}_{el}, o_{in}\right] = \frac{i}{2} \quad ; \quad \left[o^{+}_{el}, \bullet_{in}\right] = \frac{i}{2} \\ &\text{II°) case} : \left[\bullet^{+}_{in}, o_{el}\right] = \frac{i}{2} \quad ; \quad \left[o^{+}_{in}, \bullet_{el}\right] = \frac{i}{2} \end{aligned} \tag{63}$$

The other cases (see Appendix in [4]) derive by the property that a possible global exchange (• ⟷ o) doesn't change the commutators of the **(a, a+)** operators. Then it is:

$$\begin{aligned} &\text{I°) case} : \left[\bullet^{+}_{el}, \bullet_{in}\right] = \frac{i}{2} \quad ; \quad \left[o_{el}, o^{+}_{in}\right] = \frac{i}{2} \\ &\text{II°) case} : \left[\bullet^{+}_{in}, \bullet_{el}\right] = \frac{i}{2} \quad ; \quad \left[o_{in}, o^{+}_{el}\right] = \frac{i}{2} \end{aligned} \tag{64}$$

The calculation of the commutation relations is reported in [4].

3 Coupling Mechanism between two representative IQO's of two fields

Let's take two real scalar fields in the representation of the quantum operators $(\hat{a}, \hat{a}^{+})$:

$$\left\{\begin{array}{l}\hat{\Phi}_{1R}=\sum_{k}\varpi_{k}\left[\left(\hat{a}_{1k}\left(e^{-i\vec{r}\omega_{K}t+\alpha}\right)+\hat{a}^{+}_{1(-k)}\left(e^{i\vec{r}\omega_{K}t+\alpha}\right)\right)\left(e^{ikx}\right)\right] \\ \hat{\Phi}_{2R}=\sum_{k}\varpi_{k}\left[\left(\hat{a}_{2k}\left(e^{-i\vec{r}\omega_{K}t+\beta}\right)+\hat{a}^{+}_{2(-k)}\left(e^{-i\vec{r}\omega_{K}t+\beta}\right)\right)\left(e^{ikx}\right)\right]\end{array}\right\}. \tag{65}$$

The elastic coupling between the two fields (Φ_1, Φ_2) is achieved in space through a local coupling between the two respective IQO's (I_1, I_2). Denote this coupling with $\Phi_1 \oplus \Phi_2$.
A particular " **reciprocal action**" between the two IQO's (I_1, I_2) expresses this coupling:

1) This action operates a reciprocal variation of phase in two IQO's: $\Delta\varphi_1 = -\Delta\varphi_2$
2) Can happen an energy exchange (through couples of semi-quanta) between the two oscillators, but the time of exchange is uncertain as well as the phase

The eigenstates scheme of the $\mathbf{r} = (\mathbf{r_1}+\mathbf{r_2})$ operator, relative to the physical system of the two IQO's, is:

$$\boxed{|\Phi\rangle_{(IQuO1+IQuO2)} \dashv |\Phi\rangle_{(1)} \oplus |\Phi\rangle_{(2)} \Rightarrow \left\{\begin{array}{l}|\Phi(r'=\pm1)\rangle_{(1)} \oplus |\Phi(r'=\pm1)\rangle_{(2)} \\ |\Phi(r'=\pm1)\rangle_{(1)} \oplus |\Phi(r'=\mp1)\rangle_{(2)}\end{array}\right.} \tag{66}$$

The first combination is with a "non-zero eigenvalue" of operator (**r**), while the second is with a "zero eigenvalue". We examine the two possible combinations of the "non-zero eigenvalue" of (**r**): $(|\Phi_1(\pm1)\rangle) \oplus (|\Phi_2(\pm1)\rangle)$

First case: $(|\Phi_1(+1)\rangle) \oplus (|\Phi_2(+1)\rangle)$

Once reaching the adaptation (for ex. $(r_1)' = (r_2)' = +1$), we'll have (for simplicity we set $\alpha = \beta$ in eq. (65))

$$\left\{\begin{array}{l}\hat{\Phi}_{1R}=\sum_{k}\varpi_{k}\left[\left(\hat{a}_{1k}e^{-i\omega_{K}t}+\hat{a}^{+}_{1(-k)}e^{i\omega_{K}t}\right)\left(e^{ikx}\right)\right] \\ \hat{\Phi}_{2R}=\sum_{k}\varpi_{k}\left[\left(\hat{a}_{2k}e^{-i\omega_{K}t}+\hat{a}^{+}_{2(-k)}e^{i\omega_{K}t}\right)\left(e^{ikx}\right)\right]\end{array}\right\} \tag{67}$$

This particular "coupling", denoted with the new algebraic element $\oplus$, consists of a particular sum between two mathematical elements (in this case two field operators) which operates in the following way:

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = \\ = \left\{ \sum_k \varpi_k \left[\left(\hat{a}_{1k} \left(e^{-i\hat{r}\omega_K t + \alpha} \right) + \hat{a}^+_{1(-k)} \left(e^{i\hat{r}\omega_K t + \alpha} \right) \right) \left(e^{ikx} \right) \right] \right\} \oplus \\ \oplus \left\{ \sum_k \varpi_k \left[\left(\hat{a}_{2k} \left(e^{-i\hat{r}\omega_K t + \beta} \right) + \hat{a}^+_{2(-k)} \left(e^{-i\hat{r}\omega_K t + \beta} \right) \right) \left(e^{ikx} \right) \right] \right\} \tag{68}$$

Following

$$\left(\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} \right)_{|r'>} = \\ = \left[\sum_k \varpi_k \left(\hat{a}_{1k} e^{-i\omega_K t} + \hat{a}^+_{1(-k)} e^{i\omega_K t} \right) \left(e^{ikx} \right) \right] \oplus \left[\sum_k \varpi_k \left(\hat{a}_{2k} e^{-i\omega_K t} + \hat{a}^+_{2(-k)} e^{i\omega_K t} \right) \left(e^{ikx} \right) \right] = \\ = \left[\sum_k \varpi_k \left(\hat{a}_{1k} e^{-i\omega_K t} + \hat{a}^+_{1(-k)} e^{i\omega_K t} \right) \left(e^{ikx} \right) \right] \left(e^{i\varphi_1} \right) + \left(e^{i\varphi_2} \right) \left[\sum_k \varpi_k \left(\hat{a}_{2k} e^{-i\omega_K t} + \hat{a}^+_{2(-k)} e^{i\omega_K t} \right) \left(e^{ikx} \right) \right] \tag{69}$$

setting $\varphi_1 = -\varphi_2 = -\varphi$ and $\varphi = (\pi/4)$, it follows:

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = \left\{ \sum_k \varpi_k \left[\left(\hat{a}_{1k} + \hat{a}_{2k} e^{i\pi/2} \right) \left(e^{-i\pi/4} \right) \left(e^{-i\omega_K t} \right) + \left(\hat{a}^+_{1(-k)} + \hat{a}^+_{2(-k)} e^{i\pi/2} \right) \left(e^{-i\pi/4} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} \\ = e^{-i\pi/4} \cdot \left\{ \sum_k \varpi_k \left[\left(\hat{a}_k \right) \left(e^{-i\omega_K t} \right) + \left(\hat{b}^+_{(-k)} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} \tag{70}$$

where

$$\left(\hat{a}_k \right) = \left(\hat{a}_{1k} + i\hat{a}_{2k} \right) \; ; \; \left(\hat{b}^+_{(-k)} \right) = \left(\hat{a}^+_{1(-k)} + i\hat{a}^+_{2(-k)} \right) \tag{71}$$

From the $(\mathbf{a}, \mathbf{b}^+)$ form of the operators, we also obtain that

$$\left\{ \left(\hat{a}^+_k \right) = \left(\hat{a}^+_{1k} - i\hat{a}^+_{2k} \right) ; \left(\hat{b}_{(-k)} \right) = \left(\hat{a}_{1(-k)} - i\hat{a}_{2(-k)} \right) \right\} \tag{72}$$

Note that the field $[\Phi = (\Phi_1 \oplus \Phi_2)]$ is perfectly coincident with the Klein-Gordon's complex field (K-G). One can think that (see (70)) the field $[\Phi = (\Phi_1 \oplus \Phi_2)]$ is a non-local aspect (or non-separate) of the two states $\Psi_+ (\mathbf{a}, \mathbf{a}^+)$ and $\Psi_- (\mathbf{b}, \mathbf{b}^+)$ in the case of the two (K-G) fields having electric charge. Summarising we have :

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} \equiv \hat{\Psi}_C (\hat{a}, \hat{b}^+) = (\hat{\Psi}_+ \cap \hat{\Psi}_-) \tag{73}$$

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} \equiv \hat{\Psi}_C (\hat{a}^+, \hat{b}) = (\hat{\Psi}_+ \cap \hat{\Psi}_-) \tag{74}$$

where the last relation denotes the "non-separation" state.

Note that the (⊕)-operation coincides with a multiplication of matrices; namely, we'll have:

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} => ((\hat{\Phi}_{1R})) \oplus ((\hat{\Phi}_{2R})) = \begin{pmatrix} \hat{\Phi}_{1R} \\ 1 \end{pmatrix} \cdot \left[\begin{pmatrix} e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \hat{\Phi}_{2R} \end{pmatrix} \right] \tag{75}$$

To get an independent and separate exhibition of Ψ_+ and Ψ_-, we must have an eventual uncoupling, caused by an external agent-field. However, this possibility can be implemented via the coupling [$\Phi = (\Phi_1 \oplus \Phi_2)$] of the two fields belonging to "eigenvalue zero" of the global (r), that is:
$(|\Psi_1(r' = \pm 1)>) \oplus (|\Psi_2(r' = \mp 1)>)$

Second case: $(|\Psi_1(\pm 1)>) \oplus (|\Psi_2(\mp 1)>)$

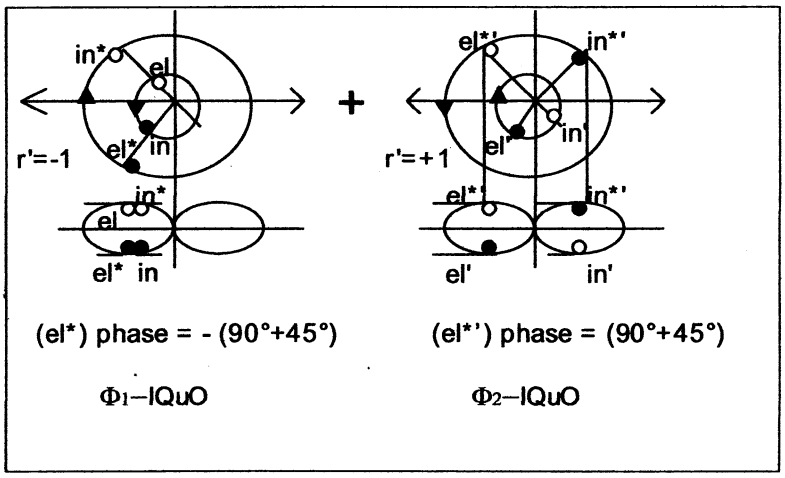


Fig. 10

with the initial configuration of the IQO in the phase $\omega t = \pm(\pi/2 + \pi/4)$.
Mathematically we have

$$\begin{aligned}
&\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = \\
&= \left[\sum_k \varpi_k \left(\hat{a}_{1k} \left(e^{i\omega_K t} \right) + \hat{a}^+_{1(-k)} \left(e^{-i\omega_K t} \right) \right) \left(e^{ikx} \right) \right] \oplus \left[\sum_k \varpi_k \left(\hat{a}_{2k} \left(e^{-i\omega_K t} \right) + \hat{a}^+_{2(-k)} \left(e^{i\omega_K t} \right) \right) \left(e^{ikx} \right) \right] = \\
&= \left[\sum_k \varpi_k \left(\hat{a}_{1k} \left(e^{i\omega_K t} \right) + \hat{a}^+_{1(-k)} \left(e^{-i\omega_K t} \right) \right) \left(e^{ikx} \right) \right] e^{i\varphi_1} + e^{i\varphi_2} \left[\sum_k \varpi_k \left(\hat{a}_{2k} \left(e^{-i\omega_K t} \right) + \hat{a}^+_{2(-k)} \left(e^{i\omega_K t} \right) \right) \left(e^{ikx} \right) \right] = \\
&= \left[\sum_k \varpi_k \left(\hat{a}_{1k} \left(e^{i\omega_K t + \varphi_1} \right) + \hat{a}^+_{1(-k)} \left(e^{-i\omega_K t + \varphi_1} \right) \right) \left(e^{ikx} \right) + \sum_k \varpi_k \left(\hat{a}_{2k} \left(e^{-i\omega_K t + i\varphi_2} \right) + \hat{a}^+_{2(-k)} \left(e^{i\omega_K t + i\varphi_2} \right) \right) \left(e^{ikx} \right) \right]
\end{aligned} \tag{77}$$

Setting $\varphi_1 = -\varphi_2 = -\varphi$, it follows

$$\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = \left\{ \sum_k \varpi_k \left[\left(\hat{a}^+_{1(-k)} \left(e^{-i\varphi} \right) + \hat{a}_{2k} \left(e^{i\varphi} \right) \right) \left(e^{-i\omega_K t} \right) + \left(\hat{a}_{1k} \left(e^{-i\varphi} \right) + \hat{a}^+_{2(-k)} \left(e^{i\varphi} \right) \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\}$$

$$= \left\{ \sum_k \varpi_k \left[\left(\hat{a}^+_{1(-k)} + \hat{a}_{2k} \left(e^{2i\varphi} \right) \right) \left(e^{-i\varphi} \right) \left(e^{-i\omega_K t} \right) + \left(\hat{a}_{1(k)} + \hat{a}^+_{2(-k)} \left(e^{2i\varphi} \right) e^{2i\varphi} \right) \left(e^{-i\varphi} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} \tag{78}$$

while, setting $\varphi = (\pi/4)$, it follows :

$$\begin{aligned}
&\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = \\
&= \left\{ \sum_k \varpi_k \left[\left(\hat{a}^+_{1(-k)} + \hat{a}_{2k} \left(e^{i\pi/2} \right) \right) \left(e^{-i\pi/4} \right) \left(e^{-i\omega_K t} \right) + \left(\hat{a}_{1(k)} + \hat{a}^+_{2(-k)} \left(e^{i\pi/} \right) e^{i\pi/2} \right) \left(e^{-i\pi/4} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} = \\
&= \left(e^{-i\pi/4} \right) \cdot \left\{ \sum_k \varpi_k \left[\left(\hat{a}^+_{1(-k)} + i\hat{a}_{2k} \right) \left(e^{-i\omega_K t} \right) + \left(\hat{a}_{1(k)} + i\hat{a}^+_{2(-k)} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} = \\
&= \left(e^{-i\pi/4} \right) \cdot \left\{ \sum_k \varpi_k \left[\left(\hat{\alpha}_{(k,-k)} \right) \left(e^{-i\omega_K t} \right) + \left(\hat{\beta}_{(k,-k)} \right) \left(e^{i\omega_K t} \right) \right] \left(e^{ikx} \right) \right\} = \\
&= \hat{\Psi}_1(\hat{\alpha}) + \hat{\Psi}_2(\hat{\beta})
\end{aligned} \tag{79}$$

An important result here is: **the fields (Ψ(α), Ψ(β)) are expressed respectively by IQO with operators of annihilation and creation having the same direction of the phase rotation.** The hamiltonian operator **H** of the physical system is established, as usual, by the field operator and his conjugated (setting $\beta = \gamma^+$)

$$\begin{cases} \hat{\Psi} = \left\{ \sum_k \varpi_k \left[\left(\hat{\alpha}_{(k,-k)} \right) e^{-i\omega_K t} + \left(\gamma^+_{(k,-k)} \right) e^{i\omega_K t} \right] e^{ikx} \right\} \\ \hat{\Psi}^+ = \left\{ \sum_k \varpi_k \left[\left(\hat{\gamma}_{(k,-k)} \right) e^{-i\omega_K t} + \left(\hat{\alpha}^+_{(k,-k)} \right) e^{i\omega_K t} \right] e^{-ikx} \right\} \end{cases} \tag{80}$$

thus obtaining

$$\hat{H} = \sum_{-\infty}^{\infty} \hbar\omega_k \left(\hat{\alpha}^+_k \hat{\alpha}_k + \gamma^+_k \gamma_k + 1 \right) \tag{81}$$

By using the semi–quanta structure (see (55)) it follows:

$$\begin{aligned}
&\hat{\Phi}_{1R} \oplus \hat{\Phi}_{2R} = e^{-i\pi/4} \cdot \left\{ \sum_k \varpi_k \left[\left(\hat{a}^+_{1(-k)} + i\hat{a}_{2k} \right) e^{-i\omega_K t} + \left(\hat{a}_{1(k)} + i\hat{a}^+_{2(-k)} \right) e^{i\omega_K t} \right] e^{ikx} \right\} = \\
&= e^{-i\pi/4} \cdot \left\{ \left[\left(\hat{\bullet}^+_{1el} - i\hat{o}^+_{1in} \right)_{-k} + i\left(\hat{\bullet}_{2el} + i\hat{o}_{2in} \right)_k \right) e^{-i\omega_K t} + \left(\left(\hat{o}_{1el} + i\,\hat{\bullet}_{1in} \right)_k + i\left(\hat{o}^+_{2el} - i\,\hat{\bullet}^+_{2in} \right)_{-k} \right) e^{i\omega_K t} \right] = \\
&= e^{-i\pi/4} \cdot \left\{ \sum_k \varpi_k \left[\left(\hat{\alpha}_{(k,-k)} \right) e^{-i\omega_K t} + \left(\gamma^+_{(k,-k)} \right) e^{i\omega_K t} \right] e^{ikx} \right\}
\end{aligned} \tag{82}$$

with

$$\begin{cases}\left(\hat{\alpha}_{(k,-k)}\right)e^{-i\omega_K t}e^{-i\pi/4}=\left((\hat{\bullet}^{+}_{1el}-i\hat{o}^{+}_{1in})_{-k}+i(\hat{\bullet}_{2el}+i\hat{o}_{2in})_{k}\right)e^{-i\omega_K t}e^{-i\pi/4}\\ \left(\gamma^{+}_{(k,-k)}\right)e^{i\omega_K t}e^{-i\pi/4}=\left((\hat{o}_{1el}+i\,\hat{\bullet}_{1in})_{k}+i(\hat{o}^{+}_{2el}-i\,\hat{\bullet}^{+}_{2in})_{-k}\right)e^{i\omega_K t}e^{-i\pi/4}\end{cases} \tag{83}$$

We can summarise, using (81):

$$\begin{cases}\hat{\alpha}=\hat{c}^{+}_{1}+i\hat{c}_{2}\\ \hat{\gamma}^{+}=\hat{d}_{1}+i\hat{d}^{+}_{2}\end{cases} \tag{89}$$

where the (c) operators and (d) satisfy the usual commutation rules. Using (see Fig.(10)) the initial configuration of the IQO with phase $\omega t = \pm(\pi/2 + \pi/4)$, one obtains :

$$\begin{cases}\left(\hat{\alpha}_{(k,-k)}\right)e^{-i\omega_K t}e^{-i\pi/4}=\left((\hat{\bullet}^{+}_{1el}e^{-i\pi}+\hat{o}^{+}_{1in}e^{-i3\pi/2})_{(cl,-k)}+(\hat{\bullet}_{2el}e^{-i\pi/2}+\hat{o}_{2in})_{(cl,k)}\right)=(\hat{\alpha}(t^{*}))^{cl}_{(k,-k)}\\ \left(\gamma^{+}_{(k,-k)}\right)e^{i\omega_K t}e^{-i\pi/4}=\left((\hat{o}_{1el}e^{i\pi/2}+\hat{\bullet}_{1in}e^{i\pi})_{(\bar{cl},k)}+(\hat{o}^{+}_{2el}e^{i\pi}+\hat{\bullet}^{+}_{2in}e^{i\pi/2})_{(\bar{cl},-k)}\right)=(\gamma^{+}(t^{*}))^{\bar{cl}}_{(k,-k)}\end{cases}$$
(90)

4 Electrically charged IQO

The state of superposition of all the possible configurations of the combination of two eigenstates $|\alpha_r>$ and $|\gamma^{+}_{r}>$ is:

$$|\Psi> \equiv [(|\alpha_1>+|\gamma^{+}_{1}>)+(|\alpha_2>+|\gamma^{+}_{2}>)+\ldots(|\alpha_n>+|\gamma^{+}_{n}>)]$$

where the eigenstates are different in phase and in distribution of the **(•, o)** semi-quanta (see (51)). In fact, if one considers the other equivalent form of eq. (55)(30.2) exchanging **(• ⟷ o)**, it will be

$$\begin{cases}\left(\hat{\alpha}'_{(k,-k)}\right)e^{-i\omega_K t}e^{-i\pi/4}=\left((\hat{o}^{+}_{1el}e^{-i\pi}+\hat{\bullet}^{+}_{1in}e^{-i3\pi/2})_{(cl,-k)}+(\hat{o}_{2el}e^{-i\pi/2}+\hat{\bullet}_{2in})_{(cl,k)}\right)=(\hat{\alpha}'(t^{*}))^{cl}_{(k,-k)}\\ \left(\hat{\beta}'_{(k,-k)}\right)e^{i\omega_K t}e^{-i\pi/4}=\left((\hat{\bullet}_{1el}e^{i\pi/2}+\hat{o}_{1in}e^{i\pi})_{(\bar{cl},k)}+(\hat{\bullet}^{+}_{2el}e^{i\pi}+\hat{o}^{+}_{2in}e^{i\pi/2})_{(\bar{cl},-k)}\right)=(\hat{\beta}'(t^{*}))^{\bar{cl}}_{(k,-k)}\end{cases}$$
(91)

Combining the two equivalent configurations

$$\begin{aligned}&(\alpha+\alpha')+(\beta+\beta')=\\ &=\left((\hat{\bullet}^{+}_{1el}e^{-i\pi}+\hat{o}^{+}_{1in}e^{-i3\pi/2})_{(cl,-k)}+(\hat{\bullet}_{2el}e^{-i\pi/2}+\hat{o}_{2in})_{(cl,k)}\right)+\\ &+\left((\hat{o}^{+}_{1el}e^{-i\pi}+\hat{\bullet}^{+}_{1in}e^{-i3\pi/2})_{(cl,-k)}+(\hat{o}_{2el}e^{-i\pi/2}+\hat{\bullet}_{2in})_{(cl,k)}\right)+\\ &+\left((\hat{o}_{1el}e^{i\pi/2}+\hat{\bullet}_{1in}e^{i\pi})_{(\bar{cl},k)}+(\hat{o}^{+}_{2el}e^{i\pi}+\hat{\bullet}^{+}_{2in}e^{i\pi/2})_{(\bar{cl},-k)}\right)+\\ &+\left((\hat{\bullet}_{1el}e^{i\pi/2}+\hat{o}_{1in}e^{i\pi})_{(\bar{cl},k)}+(\hat{\bullet}^{+}_{2el}e^{i\pi}+\hat{o}^{+}_{2in}e^{i\pi/2})_{(\bar{cl},-k)}\right)\end{aligned} \tag{92}$$

we obtain the different possibilities of recombination between the two initial IQO's which determine, at the end, two new "outgoing" IQO's:

1) First pair of IQO couples

$$\text{I°)}\begin{cases} 1) & \begin{cases} \left(\hat{\Psi}_1\right)_{cl} = \left[\left(\hat{\bullet}^{+}_{1el}\, e^{-i\pi} + \hat{o}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{\bullet}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_1\right)_{\overline{cl}} = \left[\left(\hat{o}_{1el} e^{i\pi/2} + \hat{\bullet}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{\bullet}^{+}_{2el} e^{i\pi} + \hat{o}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \\ 2) & \begin{cases} \left(\hat{\Psi}_2\right)_{cl} = \left[\left(\hat{o}^{+}_{1el} e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{\bullet}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_2\right)_{\overline{cl}} = \left[\left(\hat{\bullet}_{1el} e^{i\pi/2} + \hat{o}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{o}^{+}_{2el} e^{i\pi} + \hat{\bullet}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \end{cases} \tag{93}$$

2) Second pair of IQO couples

$$\text{II°)}\begin{cases} 3) & \begin{cases} \left(\hat{\Psi}_3\right)_{cl} = \left[\left(\hat{\bullet}^{+}_{1el}\, e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_3\right)_{\overline{cl}} = \left[\left(\hat{\bullet}_{1el} e^{i\pi/2} + \hat{\bullet}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{o}^{+}_{2el} e^{i\pi} + \hat{o}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \\ 4) & \begin{cases} \left(\hat{\Psi}_4\right)_{cl} = \left[\left(\hat{o}^{+}_{1el} e^{-i\pi} + \hat{o}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{\bullet}_{2el}\, e^{-i\pi/2} + \hat{\bullet}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_4\right)_{\overline{cl}} = \left[\left(\hat{o}_{2el} e^{i\pi/2} + \hat{o}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{\bullet}^{+}_{2el}\, e^{i\pi} + \hat{\bullet}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \end{cases} \tag{94}$$

Looking at this relations we can affirm that:

1) the first pair corresponds to two equivalent forms of only one couple of two IQO's or

$$\text{I°)}\begin{cases} IQuO(cl) \equiv \begin{cases} \left(\hat{\Psi}_1\right)_{cl} \equiv \left[\left(\hat{\bullet}^{+}_{1el} e^{-i\pi} + \hat{o}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{\bullet}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_2\right)_{cl} \equiv \left[\left(\hat{o}^{+}_{1el} e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{\bullet}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right)_{(cl,k)}\right] \end{cases} \\ IQuO(\overline{cl}) \equiv \begin{cases} \left(\hat{\Psi}_1\right)_{\overline{cl}} \equiv \left[\left(\hat{o}_{1el} e^{i\pi/2} + \hat{\bullet}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{\bullet}^{+}_{2el} e^{i\pi} + \hat{o}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \\ \left(\hat{\Psi}_2\right)_{\overline{cl}} \equiv \left[\left(\hat{\bullet}_{1el} e^{i\pi/2} + \hat{o}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{o}^{+}_{2el} e^{i\pi} + \hat{\bullet}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \end{cases} \tag{95}$$

2) the second pair corresponds to two equivalent forms of another couple of two IQO's or

$$\text{II°)}\begin{cases} IQuO(cl) \equiv \begin{cases} \left(\hat{\Psi}_3\right)_{cl} \equiv \left[\left(\hat{\bullet}^{+}_{1el}\, e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right)_{(cl,k)}\right] \\ \left(\hat{\Psi}_4\right)_{cl} \equiv \left[\left(\hat{o}^{+}_{1el} e^{-i\pi} + \hat{o}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{\bullet}_{2el}\, e^{-i\pi/2} + \hat{\bullet}_{2in}\right)_{(cl,k)}\right] \end{cases} \\ IQuO(\overline{cl}) \equiv \begin{cases} \left(\hat{\Psi}_3\right)_{\overline{cl}} \equiv \left[\left(\hat{\bullet}_{1el} e^{i\pi/2} + \hat{\bullet}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{o}^{+}_{2el} e^{i\pi} + \hat{o}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \\ \left(\hat{\Psi}_4\right)_{\overline{cl}} \equiv \left[\left(\hat{o}_{2el} e^{i\pi/2} + \hat{o}_{1in} e^{i\pi}\right)_{(\overline{cl},k)} + \left(\hat{\bullet}^{+}_{2el}\, e^{i\pi} + \hat{\bullet}^{+}_{2in} e^{i\pi/2}\right)_{(\overline{cl},-k)}\right] \end{cases} \end{cases} \tag{96}$$

As an example, we give some possible configurations of the Ψ_3-IQO (cl.):

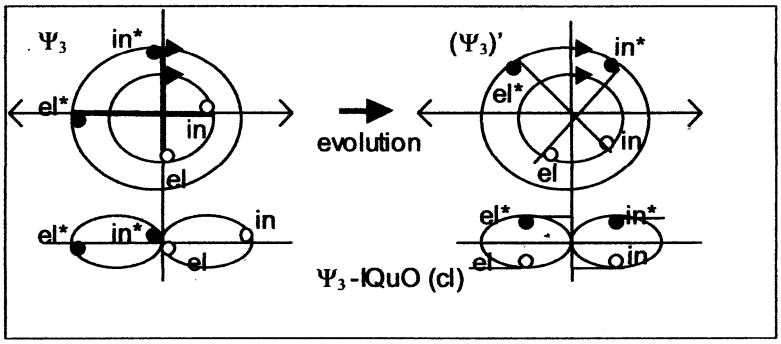


Fig.11

The anticl. IQO has a **"non-regular"** form if compared with the configurations of the cl-IQO. But we want that this IQO has a **"regular"** form as

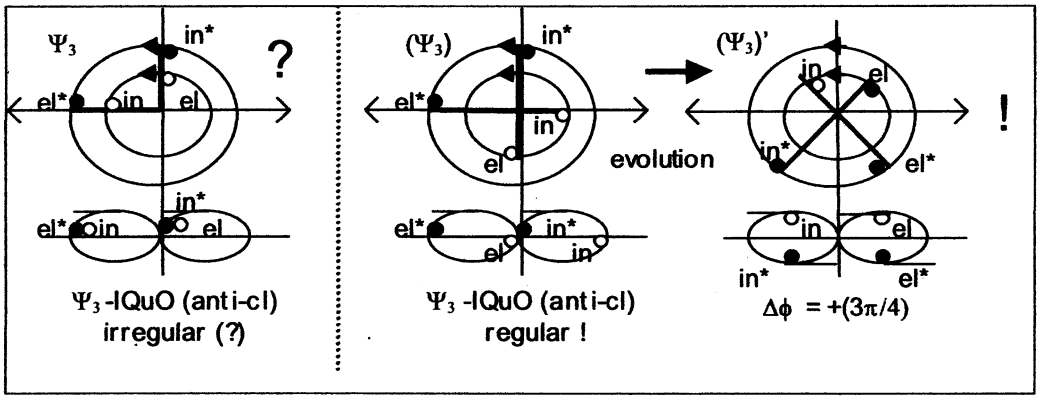


Fig.12

Note that these possible combinations of IQO could constitute some acceptable candidates for expressing a quantum field-oscillator with "electric charge": see the K-G fields: $\Psi \equiv (\Psi_+, \Psi_-)$. Expressing in matrix form these Ψ-IQO fields, we could conjecture, in regular form, the following matrices :

$$\begin{cases} \left(\left(\hat{\Psi}\right)\right)_{cl} = \begin{pmatrix} \left[\left(\hat{\bullet}_{1el}e^{-i\pi/2} + \hat{\bullet}_{1in}\right)_{(cl,-k)} \exp(ikx)\right]\exp(-i\omega_{-k}t) \\ \left[\left(\hat{o}^{+}_{1el}e^{-i\pi} + \hat{o}^{+}_{1in}e^{-i3\pi/2}\right)_{(cl,k)} \exp(ikx)\right]\exp(-i\omega_{k}t) \end{pmatrix} \\ \left(\left(\hat{\Psi}\right)\right)_{\overline{cl}} = \begin{pmatrix} \left[\left(\hat{\bullet}^{+}_{2el}e^{i\pi} + \hat{\bullet}^{+}_{2in}e^{i\pi/2}\right)_{(\overline{cl},-k)} \exp(ikx)\right]\exp(i\omega_{-k}t) \\ \left[\left(\hat{o}_{2el}e^{-i\pi/2} + \hat{o}_{2in}\right)_{(\overline{cl},k)} \exp(ikx)\right]\exp(i\omega_{k}t) \end{pmatrix} \end{cases} \qquad (97)$$

[where the k indices are adapted to the Ψ-IQO configurations (see figures 11 and 12): with (± k) we mean that the semi-quanta go in the direction of the ± x axis.]

Besides we have changed the phase of the (o_{el}, o_{in}) couple of the anti-clockwise IQO, to make it **"regular"** in a configuration like the clockwise one.

Observe that if in the $(\Phi_1 \oplus \Phi_2)$ we had executed a phase shift ($\varphi_1 = +45°$, $\varphi_2 = -45°$) opposed to that executed in the first coupling procedure ($\varphi_1 = -45°$, $\varphi_2 = +45°$) , we would have got the Ψ_1 clockwise IQO as irregular, and the Ψ_2 anticlockwise one as regular: $(\Phi_1 \oplus \Phi_2) \rightarrow (\Psi_{1(irreg)} + \Psi_{2(reg)})$.
One can understand, then, that something has to happen during the $\Phi_1 \oplus \Phi_2$ coupling (or immediately after); which, operating a phase shift on an irregular Ψ_1 , transforms this into a regular form.
A possibility would be that of a coupling with a third Φ_3-IQO, such that one of the two (Ψ_1) acquires a regular form to the detriment of the third (Φ_3). One may affirm that for any Ψ- IQO with irregular form, it must exist an external field Φ which, acting locally on the phase of Ψ, adjusts it the IQO form. It may happen also the contrary: namely, a Φ_3 external field could alter the regular form of the Ψ-IQO's. Thus, we will say that around the field Ψ there is always a Φ-agent field, which induces in it phase variations and configuration. This appearance, as we know from the literature, is analogous to the ne described by quantum mechanics where every electric charge is "dressed" by fields of virtual photons! Examining the coupling between a Ψ-IQO, with non-regular form, and a Φ-IQO, we demonstrate easily, at the end, that the Φ'-IQO outgoing is non-regular, while the Ψ'-IQO outgoing is in regular form. Now, we prove that a Ψ IQO-field has electric charge. We calculate the electric charge of the (Ψ_{cl}) field in the semi-quanta representation:

$$\hat{\Psi}_{cl} \equiv \left[\left(\hat{\bullet}^{+}_{1el}\, e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right)_{(cl,-k)} + \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right)_{(cl,k)}\right] \tag{98}$$

in matrix form (see eq. 7.4)

$$\left(\left(\hat{\Psi}_{cl}\right)\right) \equiv \begin{pmatrix} \left(\hat{o}_{2el} e^{-i\pi/2} + \hat{o}_{2in}\right) \\ \left(\hat{\bullet}^{+}_{1el}\, e^{-i\pi} + \hat{\bullet}^{+}_{1in} e^{-i3\pi/2}\right) \end{pmatrix} \tag{99}$$

where the indices [(cl, k), (cl,-k)] have been omitted.
The distribution of the operator pairs of semi-quanta is not arbitrary, because, as will be shown later, the dispositions of the matrix elements is related to the rotation direction of the phase.
For calculating Q it's necessary to consider also the Ψ^{+} hermitian field

$$\left(\left(\hat{\Psi}^{+}_{\bar{cl}}\right)\right) \equiv \begin{pmatrix} \left(\hat{o}^{+}_{el} e^{i\pi/2} + \hat{o}^{+}_{in}\right) \\ \left(\hat{\bullet}_{el}\, e^{i\pi} + \hat{\bullet}^{i3\pi/2}_{in}\right) \end{pmatrix} \tag{100}$$

but we ask that the conjugation operation of Ψ contains the exchange ($\bullet \leftrightarrow o$). Therefore, we'll have:

$$\left(\left(\hat{\Psi}^{+}_{\bar{cl}}\right)\right) \equiv \begin{pmatrix} \left(\hat{\bullet}^{+}_{el}\, e^{i\pi/2} + \hat{\bullet}^{+}_{in}\right) \\ \left(\hat{o}_{el} e^{i\pi} + \hat{o}^{i3\pi/2}_{in}\right) \end{pmatrix} \tag{101}$$

Using the well known definition of the electric charge [1] and the commutation relations with semi-quanta (see Appendix in [4]) it then follows that:

$$\begin{aligned}
&Q_{cl} = \int\left[\left(\left(\hat{\Psi}^{+}_{(-)}\right)\right)\hat{\sigma}_3\left(\left(\hat{\Psi}_{(-)}\right)\right)\right]dV = \int\begin{pmatrix}\left(\hat{\bullet}^{+}_{el}\, e^{i\pi/2} + \hat{\bullet}^{+}_{in}\right)\\ \left(\hat{o}_{el} e^{i\pi} + \hat{o}^{i3\pi/2}_{in}\right)\end{pmatrix}\begin{pmatrix}\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right)\\ -\left(\hat{\bullet}^{+}_{el}\, e^{-i\pi} + \hat{\bullet}^{+}_{in} e^{-i3\pi/2}\right)\end{pmatrix} dV \equiv \\
&Q = \left[\left(\left(\hat{\bullet}^{+}_{el}\, e^{i\pi/2}\right) + \hat{\bullet}^{+}_{in}\right)\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right) - \left(\hat{o}_{el} e^{i\pi} + \hat{o}^{i3\pi/2}_{in}\right)\left(\hat{\bullet}^{+}_{el}\, e^{-i\pi} + \hat{\bullet}^{+}_{in} e^{-i3\pi/2}\right)\right] = \\
&\left(i\,\hat{\bullet}^{+}_{el}\,\hat{o}_{in} - i\,\hat{\bullet}^{+}_{el}\, e^{-i\pi}\hat{o}^{i3\pi/2}_{in}\right) + \left(\hat{\bullet}^{+}_{in}\,\hat{o}_{el} e^{-i\pi/2} - \hat{\bullet}^{+}_{in} e^{-i3\pi/2}\hat{o}_{el} e^{i\pi}\right) \\
&\quad = \left[\hat{\bullet}^{+}_{el}, \hat{o}_{el}\right] + \left[\hat{\bullet}^{+}_{in}, \hat{o}_{in}\right] + i\left[\hat{\bullet}^{+}_{el}, \hat{o}_{in}\right] + i\left[\hat{o}_{el}, \hat{\bullet}^{+}_{in}\right] = -1
\end{aligned} \tag{102}$$

where $(\Psi_{-}) = (\Psi_{cl})$ and σ_3 is a Pauli's matrix. We calculate also the electric charge of the $(\Psi_{anti\text{-}cl})$ regular IQO. In matrix form (see (97)) it is

$$\left(\left(\hat{\Psi}\right)\right)_{\overline{cl}} = \begin{pmatrix}\left[\left(\hat{\bullet}^{+}_{el} e^{i\pi} + \hat{\bullet}^{+}_{in} e^{i\pi/2}\right)\right]\\ \left[\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right)\right]\end{pmatrix} \tag{103}$$

But it is not true that

$$\left(\left(\hat{\Psi}\right)\right)_{\overline{cl}} = \begin{pmatrix}\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right)\\ \left(\hat{\bullet}^{+}_{el} e^{i\pi} + \hat{\bullet}^{+}_{in} e^{i\pi/2}\right)\end{pmatrix} \tag{104}$$

because, as we shall see later, the order of the coefficients in (Ψ_{cl}) e $(\Psi_{anti\text{-}cl})$ is connected to the the direction of phase rotation. Proceeding, it follows

$$\begin{aligned}
&Q_{\overline{cl}} = \int\left[\left(\left(\hat{\Psi}^{+}_{(+)}\right)\right)\hat{\sigma}_3\left(\left(\hat{\Psi}_{(+)}\right)\right)\right]dV = \int\begin{pmatrix}\left(\hat{o}_{el} e^{-i\pi} + \hat{o}_{in} e^{-i\pi/2}\right)\\ \left(\hat{\bullet}^{+}_{el}\, e^{i\pi/2} + \hat{\bullet}^{+}_{in}\right)\end{pmatrix}\begin{pmatrix}\left(\hat{\bullet}^{+}_{el} e^{i\pi} + \hat{\bullet}^{+}_{in} e^{i\pi/2}\right)\\ -\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right)\end{pmatrix} dV = \\
&\quad = \left[\left(\hat{o}_{el} e^{-i\pi} + \hat{o}_{in} e^{-i\pi/2}\right)\left(\hat{\bullet}^{+}_{el}\, e^{i\pi} + \hat{\bullet}^{+}_{in} e^{i\pi/2}\right) - \left(\hat{\bullet}^{+}_{el}\, e^{i\pi/2} + \hat{\bullet}^{+}_{in}\right)\left(\hat{o}_{el} e^{-i\pi/2} + \hat{o}_{in}\right)\right] = \\
&\quad = \hat{o}_{el}\,\hat{\bullet}^{+}_{el} + \hat{o}_{el}\,\hat{\bullet}^{+}_{in}\, e^{-i\pi/2} + \hat{o}_{in}\,\hat{\bullet}^{+}_{el}\, e^{i\pi/2} + \hat{o}_{in}\,\hat{\bullet}^{+}_{in} - \hat{\bullet}^{+}_{el}\,\hat{o}_{el} - \hat{\bullet}^{+}_{el}\,\hat{o}_{in} e^{i\pi/2} - \hat{\bullet}^{+}_{in}\,\hat{o}_{el} e^{-i\pi/2} - \hat{\bullet}^{+}_{in}\,\hat{o}_{in} = \\
&\quad = \left[\hat{o}_{el}, \hat{\bullet}^{+}_{el}\right] + \left[\hat{o}_{in}, \hat{\bullet}^{+}_{in}\right] + i\left[\hat{\bullet}^{+}_{in}, \hat{o}_{el}\right] + i\left[\hat{o}_{in}, \hat{\bullet}^{+}_{el}\right] = +1
\end{aligned} \tag{105}$$

Notice that, using the irregular form, the same result is obtained. In fact it is:

$$\begin{aligned}
&Q_{\overline{cl}} = \int\left[\left(\left(\hat{\Psi}^{+}_{(+)}\right)\right)\hat{\sigma}_3\left(\left(\hat{\Psi}_{(+)}\right)\right)\right]dV = \\
&\quad = \left[\left(\hat{o}_{el} e^{-i\pi} + \hat{o}_{in} e^{-i\pi/2}\right)\left(\hat{\bullet}^{+}_{el}\, e^{i\pi} + \hat{\bullet}^{+}_{in} e^{i\pi/2}\right) - \left(\hat{\bullet}^{+}_{el}\, e^{-i\pi/2} + \hat{\bullet}^{+}_{in} e^{-i\pi}\right)\left(\hat{o}_{el} e^{i\pi/2} + \hat{o}^{i\pi}_{in}\right)\right] = \\
&\quad = \left[\hat{o}_{el}, \hat{\bullet}^{+}_{el}\right] + \left[\hat{o}_{in}, \hat{\bullet}^{+}_{in}\right] + i\left[\hat{\bullet}^{+}_{in}, \hat{o}_{el}\right] + i\left[\hat{o}_{in}, \hat{\bullet}^{+}_{el}\right] = +1
\end{aligned} \tag{106}$$

This result says us that the phase shift (operated by an outside agent to regulate the IQO), does not change the sign of the electric charge, and this corresponds to the gauge invariance of the electromagnetic interactions!

In short, we can notice that: the $(\Phi_1 \oplus \Phi_2)$ coupling of two IQO's has generated two particle-IQO's with opposite electric charge: $(\Psi_{anti\text{-}el})$, (Ψ_{el}).
Note that the sign of the electric charge is connected to the direction of the phase rotation!
The process of coupling between two Φ-IQO is synthesized by

$$\hat{\Phi}_1 \oplus \hat{\Phi}_2 = \hat{\Psi}_+ + \hat{\Psi}_- \tag{107}$$

The coupling mechanism can be simplified by placing the two IQO's in the same initial configuration (see Fig 10 and Fig 14):

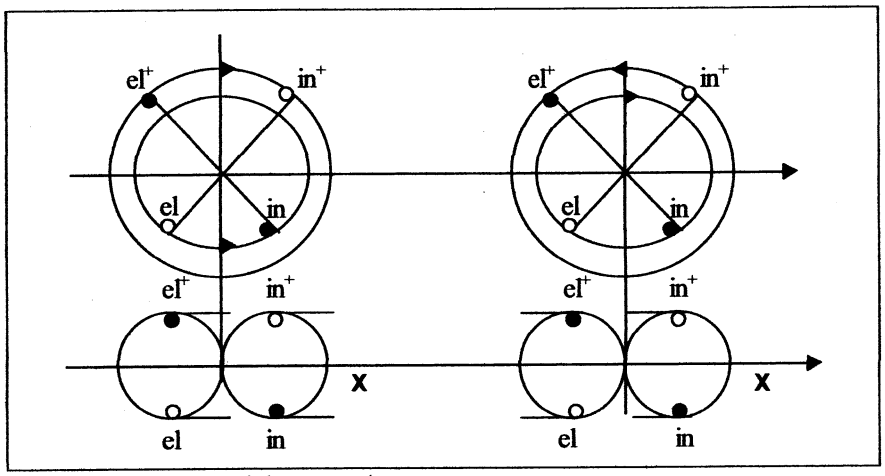


Fig. 13 : Scalar Φ-IQO

We have the following matrices

$$\left\{\left(\left(\hat{\Phi}_1\right)\right)\equiv\begin{pmatrix}\left(\hat{\bullet}^{+}_{1el}\, e^{-i(5/4)\pi}+\hat{o}^{+}_{1in}e^{-i(7/4)\pi}\right)\\ \left(\hat{o}_{1el}e^{i(5/4)\pi}+\hat{\bullet}_{1in}\, e^{i(7/4)\pi}\right)\end{pmatrix}\quad\left(\left(\hat{\Phi}_2\right)\right)\equiv\begin{pmatrix}\left(\hat{\bullet}^{+}_{2el}\, e^{i(3/4)\pi}+\hat{o}^{+}_{2in}e^{i(1/4)\pi}\right)\\ \left(\hat{o}_{2el}e^{-i(3/4)\pi}+\hat{\bullet}_{2in}\, e^{-i(1/4)\pi}\right)\end{pmatrix}\right\} \tag{108}$$

Looking at (97), we can write in general terms

$$\left\{\left(\left(\hat{\Phi}_1\right)\right)\equiv\begin{pmatrix}\left(\hat{\bullet}^{+}_{1el}\, e^{-i\alpha}+\hat{o}^{+}_{1in}e^{-i\alpha^{\circ}}\right)\\ \left(\hat{o}_{1el}e^{i\beta}+\hat{\bullet}_{1in}\, e^{i\beta^{\circ}}\right)\end{pmatrix}\quad\left(\left(\hat{\Phi}_2\right)\right)\equiv\begin{pmatrix}\left(\hat{\bullet}^{+}_{2el}\, e^{i\gamma}+\hat{o}^{+}_{2in}e^{i\gamma^{\circ}}\right)\\ \left(\hat{o}_{2el}e^{-i\rho}+\hat{\bullet}_{2in}\, e^{-i\rho^{\circ}}\right)\end{pmatrix}\right\} \tag{109}$$

where $[(\alpha^{\circ} = \alpha + \pi/2), (\beta^{\circ} = \beta + \pi/2), (\gamma^{\circ} = \gamma - \pi/2), (\rho^{\circ} = \rho - \pi/2)]$.
It is possible that it be $\Delta\varphi_1 = -\Delta\varphi_2 = 0$.

Using the matrices we obtain (see (75), (76) and (77))

$$\hat{\Phi}_1 \oplus \hat{\Phi}_2 = \begin{pmatrix} \hat{\Phi}_1 \\ 1 \end{pmatrix} \cdot \left[\begin{pmatrix} e^{-i\Delta\varphi} & 0 \\ 0 & e^{i\Delta\varphi} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \hat{\Phi}_2 \end{pmatrix} \right] = \begin{pmatrix} \hat{\Phi}_1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ \hat{\Phi}_2 \end{pmatrix} = \hat{\Psi}_\alpha + \hat{\Psi}_\beta \tag{110}$$

A possible combination of semi-quanta (s-q) of the IQO-1 with those of IQO-2 is shown in (95). If we use the matrix form (see (108)) in this combination, we can represent the coupling mechanism between the two initial IQO's (after admitting an exchange of energy ($\bullet \leftrightarrow o$)) in the following way

$$\begin{aligned} \left(\hat{\Phi}_1 \oplus \hat{\Phi}_2\right) &= \left\{\hat{\Phi}_1 \middle| \hat{\Omega} \middle| \hat{\Phi}_2\right\} = \\ &= \left\{ \begin{pmatrix} \left(\hat{\bullet}^+_{1el}\, e^{-i(5/4)\pi} + \hat{o}^+_{1in} e^{-i(7/4)\pi}\right) \\ \left(\hat{o}_{1el} e^{i(5/4)\pi} + \hat{\bullet}_{1in} e^{i(7/4)\pi}\right) \end{pmatrix} \left(\hat{\Omega}\right) \begin{pmatrix} \left(\hat{\bullet}^+_{2el}\, e^{i(3/4)\pi} + \hat{o}^+_{2in} e^{i(1/4)\pi}\right) \\ \left(\hat{o}_{2el} e^{-i(3/4)\pi} + \hat{\bullet}_{2in} e^{-i(1/4)\pi}\right) \end{pmatrix} \right\} = \\ &= \begin{pmatrix} \left(\hat{\bullet}^+_{1el}\, e^{-i(5/4)\pi} + \hat{\bullet}^+_{1in} e^{-i(7/4)\pi}\right) \\ \left(\hat{o}_{2el} e^{-i(3/4)\pi} + \hat{o}_{2in} e^{-i(1/4)\pi}\right) \end{pmatrix} + \begin{pmatrix} \left(\hat{\bullet}^+_{2el}\, e^{i(3/4)\pi} + \hat{\bullet}^+_{2in} e^{i(1/4)\pi}\right) \\ \left(\hat{o}_{1el} e^{i(5/4)\pi} + \hat{o}_{1in} e^{i(7/4)\pi}\right) \end{pmatrix} \end{aligned} \tag{111}$$

Graphically it is:

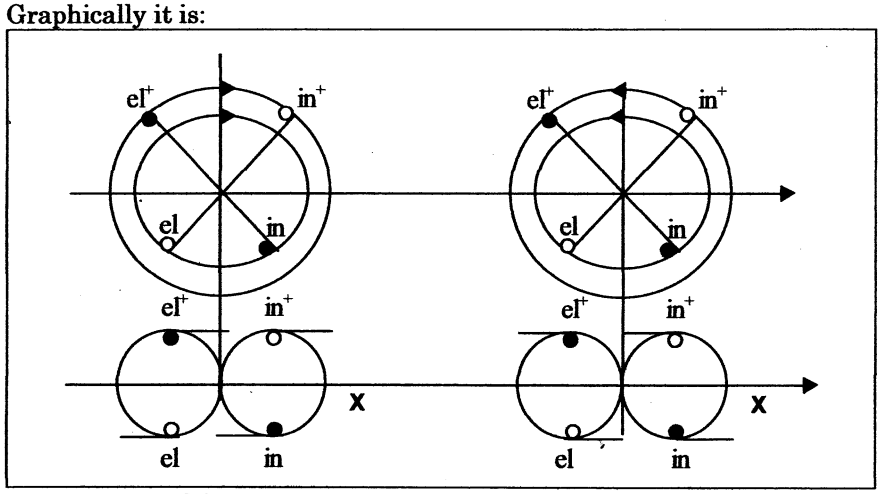


Fig. 14: Two IQO with opposite "carge"

Where the ⊕-operation is equivalent to the Ω-operation defined as:

$$\hat{\Omega} = \left(\overline{\rangle\langle} \cdot \hat{\Sigma}_{(\bullet \Leftrightarrow o)} \cdot \hat{\varphi}(0)\right) = \left[\left(\overline{\rangle\langle}\right) \left(\overline{\Sigma}(\bullet \Leftrightarrow o)\right) \begin{pmatrix} e^{-i\Delta\varphi} & 0 \\ 0 & e^{i\Delta\varphi} \end{pmatrix}_{\varphi=0} \right]$$

$$\text{with} \left\{ \begin{array}{l} \hat{\varphi}(0) = \begin{pmatrix} e^{-i\Delta\varphi} & 0 \\ 0 & e^{i\Delta\varphi} \end{pmatrix}_{\varphi=0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \equiv (\text{matrix operator of phase shift}) \\ \left(\overline{\Sigma}(\bullet \Leftrightarrow o)\right) \equiv (\text{operation of energy exchange between s-q}) \\ \left(\overline{\rangle\langle}\right) \equiv \quad (\text{operation coupling diagonally of matricial elements}) \end{array} \right\}$$

5 IQO having non-integer electric charge

The multiplets existing in the hadrons are described by three physical quantities: hypercharge (Y), isospin (T_3) and electrical charge Q. The abstract space to describe the hadrons, requires the transformation group SU (3). The generators of the group are the Gell-Mann matrices associated to the three variables (Y, T_3, Q). We focus on the matrices λ_3 and λ_8, two generators of the group [5]:

$$\left\{ \hat{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \right\} \tag{112}$$

The λ_3 matrix is the projection in SU (3) of the Pauli's matrix σ_3, giving the values of Q (see eq. 11.4) (102) in the representation of the basis eigenvectors (Ψ_+,Ψ_-). Remember that σ_3 matrix is also used to represent the projection (T_3) of isospin T in the complex space Q (2). In fact it is

$$\boxed{\begin{array}{l} \left\{ \left[\hat{T}_i\big|_{i=1,2,3} = \left(\frac{1}{2}\right) \hat{\sigma}_i\big|_{i=1,2,3} \right] \in U(2) \right. \\ \left. \left[\hat{T}_i\big|_{i=1,2,3} = \left(\frac{1}{2}\right) \hat{\lambda}_i\big|_{i=1,2,3} \right] \in U(3) \right\} \\ \text{con } \hat{T}_3 = \left(\frac{1}{2}\right) \hat{\lambda}_3 = \left(\frac{1}{2}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}} \tag{113}$$

where the matrices (T_i) (with i = 1,2,3) generate the subgroup SU(2) of SU(3). Instead with the three variables (Q,T,Y), we build the Gell-Mann matrices:

$$\boxed{\left\{ \begin{array}{l} T \Rightarrow \left[\hat{\sigma}_1 \Rightarrow \left(\hat{\lambda}_1, \hat{\lambda}_4, \hat{\lambda}_6\right) ; \; \hat{\sigma}_2 \Rightarrow \left(\hat{\lambda}_2, \hat{\lambda}_5, \hat{\lambda}_7\right) ; \hat{\sigma}_3 \Rightarrow \left(\hat{\lambda}_3\right) \right] \\ Q \Rightarrow \left[\hat{\sigma}_3 \Rightarrow \left(\hat{\lambda}_3\right) \right] \\ Y \Rightarrow \left[\left(\hat{\lambda}_8\right) \right] \end{array} \right\}} \tag{114}$$

This formal analogy between σ_i e (T_i), as between σ_3 and (T_3), cannot be casual.

Remembering that the electric charge is connected to the rotation of the phase of an IQO (as "internal" rotation), it is known that we can connect the isospin (since it has similar structure to that of the "spin") to an "internal rotation" of "something" that exists "inside" the hadrons.

So if the conservation of Q is consequent with the invariance of the direction of the phase rotation for phase shifts induced by the electromagnetic coupling between electric charge and a photon, the conservation of the isospin would be consequent to the invariance of the direction of the "internal" rotation for phase shifts induced by the strong interaction.

The same goes for the transformations between hadrons induced by weak interactions: the conservation of total isospin can derive from a global invariance relating the rotations of "something" that is "inside" each hadron participating to the interaction.
As is well known, the transformations between hadrons shows the existence of multiplets.
If T_3 differentiates the members of a sub-multiplet identified by the value of T (eg., the set of pions π), hypercharge Y differentiates the sub-groups of multiplets within a group of hadrons. The "organization" of hadron in mass degenerate multiplets and the internal transformations between hadrons proves that the hadrons are to be composed by "more elementary" particles (called quarks).
The existing "rotation" inside an hadron could well relate to quarks; if the spin of a quark regards its spatial orientation, the isospin cannot refer to ulterior additional spatial orientations. Then we could assume that the **"rotation" of isospin** should be related to the "rotation" of the internal current of "semi-quanta" that interconnect the quarks constituting a hadron (gluon current?). Also notice that the formal analogy between (T_3) and the spin of a composite particle induces to assign a value of (T_3) also to quarks.
It may be recalled in fact the summative character of the isospin; then it follows that the isospin of a hadron is the sum of the values of isospin assigned to the each individual quark.
Furthermore, the correlation between T_3 and Q, leads us to admit the existence of a correlation between the direction of rotation of the phase and the one of the isospin current.
This clearly happens in individual quarks, because there are the following associations:

1) Quark (u) : $Q\ (>0) \leftrightarrow (T_3)' = (+1/2)$
2) Quark (d) : $Q\ (<0) \leftrightarrow (T_3)' = (-1/2)$

Note that Q and T3 have the same sign.

Thus, ultimately, we can assign three intrinsic " rotations" to a IQO-quark:

1) Internal phase rotation (Q sign)
2) Internal Isopin rotation (T_3 sign)
3) External Spatial rotation of spin (σ sign)

This may seem at odds with the "spatially local" aspect assigned to the elementary particle (like quarks and leptons) , in perfect agreement with relativistic quantum theory.
However, considering what already said about the rotations inside a hadron and taking into account the phenomenology of the interactions between hadrons, it can become acceptable to assume the following **Hypothesis of Structure**: although an elementary particle seems to prove to be essentially a "local unit" in space for an external observer, it may indirectly show – as suggested by many authors – the existence of an "internal" space, associated

with variations of physical quantities that can be identified as "internal" rotations; showing thus an "internal structure" of couplings of IQO.
If the isospin (T_h) assigned to a hadron identifies an internal current (semi-quanta current or gluonic current) then the isospin (T_q) assigned to a quark identically detects the existence of a semi-quanta current that identifies a "path" between elementary "internal points of oscillation", even when choosing to go on regarding a quark as an "oscillating" single physical system.
A quark, even when maintaining the external aspect of a quantum of energy at a space point, could instead be constituted by an internal structure of "virtual" IQO's which we call Sub-IQO. This "virtual" internal structure of a quark has a descriptive index that we individuate in the ipercharge Y. The latter, as well as being introduced to differentiate the hadron multiplets, is assigned to each individual quark, reinforcing the idea of a sub-structure of its related to different configurations of coupling of Sub-IQO's that constitute it.
Another support for this **Hypothesis of Structure** is identifiable in the correlations between the "internal" rotations (those of phase and isospin) during the hadron interactions.
It was noted that Q and T_3 have the same algebraic sign; this implies that **the isospin current between Sub-IQO constituents of a quark follows the same "direction" of rotation of the phase of each Sub-IQO. It's evident that the structure of the Sub-IQO's (think of the Y ipercharge) has to be compatible with the correlation** ($Q \leftrightarrow T_3$). It follows that it has to exist a connection ($Q \leftrightarrow T_3 \leftrightarrow Y$).
The relation by Gell-Mann expresses this connection: $[Y = 2\,(Q - T_3)]$
The correlation ($Q \leftrightarrow T_3 \leftrightarrow Y$) is made manifest both in the correlation between the matrices λ_3 e λ_8, as we are going to see, and in the combination of the respective generators during interactions. This is more evident for the rotation of phase (Q) and rotation of current (T_3):

$Q \leftrightarrow J_{(Q)} \leftrightarrow | J_{(Q)}'>$
$T_3 \leftrightarrow J_{(T)} \leftrightarrow | J_{(T3)}'>$

Remember that when one composes rotations (generators - angular momenta (J)) then he has to use the Clebesch-Gordan coefficients. Therefore, the correlations between internal rotations can be expressed by the Clebesch – Gordan coefficients.
Recall, for example, the reaction **$[\pi^- + p \rightarrow \pi^- + p]$**, where there are evident the following amplitudes of transition about the isospin:

$$\begin{array}{ll} \pi^- + p & \Rightarrow \quad \pi^- + p \\ |1,0\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle & \Rightarrow \left(\sqrt{\frac{1}{3}}\right)\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \left(\sqrt{\frac{2}{3}}\right)\left|\frac{1}{2},-\frac{1}{2}\right\rangle \end{array} \qquad (115)$$

Relevant information is obtained from the corresponding probability amplitudes of the S matrix:

$$A_{(\pi,p)} = \left[\left(\sqrt{\frac{1}{3}}\right)\left(\sqrt{\frac{1}{3}}\right)\right]\left\langle \frac{3}{2},-\frac{1}{2}\middle|S\middle|\frac{3}{2},-\frac{1}{2}\right\rangle + \left[\left(\sqrt{\frac{2}{3}}\right)\left(\sqrt{\frac{2}{3}}\right)\right]\left\langle \frac{1}{2},-\frac{1}{2}\middle|S\middle|\frac{1}{2},-\frac{1}{2}\right\rangle = \left(\frac{1}{3}\right)A_{(1)} + \left(\frac{2}{3}\right)A_{(2)} \tag{116}$$

We do not think it is a casual case that the coefficients of the "probability" amplitudes coincide with the two possible values of the electric charge of quarks.

Underlining the correlation of the sign of Q (direction of phase rotation) with the sign of T (direction of current rotation) and with value of Y (structure by couplings), one derive that the electric charge values are connected with the structure of quarks. It's evident that this connection determines in hadron interactions the various probabilities of combination of isospin couplings, as well as the probabilities of the electric charge values of the quarks. This also suggests that the non-integer value of the charge of a quark is expression of probabilistic aspects associated to the structure of couplings of the Sub-IQO's which compose the quark. **If the direction of the phase rotation of a particle-IQO gives to us the sign of the electric charge, its value can instead be placed in relation with the probability that a photon detects the quanta of the charged particle with a particular structure of sub-IQO's.** If this probability has value equal to one for an electron, the probability in the quarks is not an integer value. **If inside a quark there are more sub-IQO's, the photon in coupling with one of sub-IQO components cannot find the representative quantum of the quark.** For the definition of statistical probability, if a quark cannot be seen by a single photon, then it will be not seen by an infinite number of photons. In other words, a quark would be invisible to a single photon.

However, if the single quark cannot be seen, a hadron composed by more quarks can be detected if the sum of the probability of finding the quantum has value of unity, or the sum of the electric charges of the quarks of an hadron reaches the value of a unity. This is because two or three quarks bound become a single unit (or a single center, diffuser) for couplings with photons. The information of the non-integer value of the electric charge of a quark can also be found in the Gell-Mann matrix λ_8, which is expression of the connection (Q ⟷ Y). In fact, it is no casual coincidence that the matrix λ_8 does precisely commute with the λ_3 (see σ_3). We recall the correlation between Y and the matrix λ_8:

$$\hat{Y} = \left(\frac{1}{\sqrt{3}}\right)\hat{\lambda}_8 \quad \Rightarrow \quad \hat{Y} = \left(\frac{1}{3}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{117}$$

The eigenvalues of Y are:

$$\hat{Y}' \Rightarrow \left\{ \begin{vmatrix} [(1/3)-\lambda'] & 0 & 0 \\ 0 & [(1/3)-\lambda'] & 0 \\ 0 & 0 & [(-2/3)-\lambda'] \end{vmatrix} = 0 \right\} \Rightarrow \left\{ [(1/3-\lambda')^2][(-2/3)-\lambda'] \right\} = 0$$

$$Y' = \begin{Bmatrix} 1/3 \\ 1/3 \\ -2/3 \end{Bmatrix} \qquad (118)$$

Also note here that the eigenvalues of Y are related to the values of electric charge Q. Obviously everything is perfectly in accord with the relation: [Y = 2 (Q-T_3)].

Given the correlation between the charge Q and isospin T_3 and Y, the state of a hadron can be regarded as a state composed of simultaneous eigenstates of "rotations" (induced by Q and T_3) and Y. Therefore we can consider the matrix product $\hat{\lambda}_8 = (\hat{\lambda}_3)(\hat{Y})$

$$\hat{\lambda}_3 = \hat{\lambda}_3 \hat{Y} = \left(\frac{1}{3}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \left(\frac{1}{3}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} (1/3) & 0 & 0 \\ 0 & -(1/3) & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (119)$$

In conclusion, the matrix $\hat{\lambda}_3$ can tell what values are allowed for the charge Q of a quark.

To calculate these values we resort to the form of a matrix IQO.

In Sect. 4, the coupling between two IQO "neutral" in the direction of the phase rotation but with opposite eigenvalues of the operator (r), $[(r')_1 = +1, (r')_2 = -1]$, has produced a pair of integer IQO electric charges $[(Q')_1 = +1, (Q')_2 = -1]$. The quarks leads one to seek a possible coupling between Φ-IQO pairs (electrically neutral) that produce outgoing Ψ-IQO pairs having non-integer electric charge. A different form of the representative matrix for an IQO is to be found in such a way that it be possible to find, by applying 11.4 (102) and (103) 12.4, non-integer values of the eigenvalues of Q. Now we find the column matrix (Ψ) which is representative of a IQO-quark with electric charge, to apply the relationship defined by (102)11.4. It is obvious that with 3x3 matrices one must have a matrix column (Ψ) with three elements (1x3 matrix).

Note that the matrix Y is the only G-M matrix that by operating on a column matrix (1x3) keeps unchanged the number 3 of its elements. To having an IQO with three elements matrix, it is necessary to resort simultaneously to coupling between three Φ_i -IQO. We put:

$$\hat{\Phi}_1 \oplus \hat{\Phi}_2 \oplus \hat{\Phi}_3 = \hat{\Psi}_+ + \hat{\Psi}_- \qquad (120)$$

In matrix form (see (109), (110), (111), (112)) we conjecture that:

$$\left\{ \hat{\Psi}_{cl} = \begin{pmatrix} \left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) \\ \left(\hat{\bullet}^+_{el} e^{-i\alpha} + \hat{o}^+_{in} e^{-i\alpha^\circ}\right) \\ \left(\hat{o}_{el} e^{-i\sigma} + \hat{\bullet}_{in} e^{-i\sigma^\circ}\right) \end{pmatrix} \quad , \quad \hat{\Psi}_{\bar{cl}} = \begin{pmatrix} \left(\hat{o}^+_{el} e^{i\gamma} + \hat{\bullet}^+_{in} e^{i\gamma^\circ}\right) \\ \left(\hat{\bullet}_{el} e^{i\beta} + \hat{o}_{in} e^{i\beta^\circ}\right) \\ \left(\hat{\bullet}^+_{el} e^{i\varepsilon} + \hat{o}^+_{in} e^{i\varepsilon^\circ}\right) \end{pmatrix} \right\} \tag{121}$$

We can now calculate the electric charge:

$$Q_{cl} = \left(\left(\hat{\Psi}^+\right)\right)_{cl} \hat{\lambda}_3 \left(\left(\hat{\Psi}\right)\right)_{cl} = \begin{pmatrix} \left(\hat{\bullet}^+_{el} e^{i\rho} + \hat{o}^+_{in} e^{i\rho^\circ}\right) \\ \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right) \\ \left(\hat{\bullet}^+_{el} e^{i\sigma} + \hat{o}^+_{in} e^{i\sigma^\circ}\right) \end{pmatrix} \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) \\ \left(\hat{\bullet}^+_{el} e^{-i\alpha} + \hat{o}^+_{in} e^{-i\alpha^\circ}\right) \\ \left(\hat{o}_{el} e^{-i\sigma} + \hat{\bullet}_{in} e^{-i\sigma^\circ}\right) \end{pmatrix} \tag{122}$$

$$\begin{aligned} Q_{cl} &= (1/3) \begin{pmatrix} \left(\hat{\bullet}^+_{el} e^{i\rho} + \hat{o}^+_{in} e^{i\rho^\circ}\right) \\ \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right) \\ \left(\hat{o}^+_{el} e^{i\sigma} + \hat{\bullet}^+_{in} e^{i\sigma^\circ}\right) \end{pmatrix} \begin{pmatrix} \left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) \\ -\left(\hat{\bullet}^+_{el} e^{-i\alpha} + \hat{o}^+_{in} e^{-i\alpha^\circ}\right) \\ 0 \end{pmatrix} = \\ &= (1/3) \left\{ \left(\hat{\bullet}^+_{el} e^{i\rho} + \hat{o}^+_{in} e^{i\rho^\circ}\right) \left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) - \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right) \left(\hat{\bullet}^+_{el} e^{-i\alpha} + \hat{o}^+_{in} e^{-i\alpha^\circ}\right) \right\} = \\ &= (1/3) \left\{ \begin{array}{l} \left[\hat{\bullet}^+_{el}, \hat{o}_{el}\right] + \left[\hat{o}^+_{in}, \hat{\bullet}_{in}\right] + \\ + \left(\hat{\bullet}^+_{el} \hat{\bullet}_{in} e^{i(\rho - \rho^\circ)} - \hat{\bullet}_{in} \hat{\bullet}^+_{el} e^{i(\alpha^\circ - \alpha)}\right) + \left(\hat{o}^+_{in} \hat{o}_{el} e^{i(\rho^\circ - \rho)} - \hat{o}_{el} \hat{o}^+_{in} e^{i(\alpha - \alpha^\circ)}\right) \end{array} \right\} \\ &= (1/3) \left\{ \left[\hat{\bullet}^+_{el}, \hat{\bullet}_{in}\right] e^{i(\pi/2)} + \left[\hat{o}^+_{in}, \hat{o}_{el}\right] e^{-i(\pi/2)} \right\} = (1/3) \left\{ i\left(\frac{i}{2}\right) - i\left(\frac{-i}{2}\right) \right\} = -(1/3) \end{aligned}$$

(123)

where $[(\alpha^\circ = \alpha + \pi/2), (\rho^\circ = \rho - \pi/2)]$.

It is easily calculated that $Q_{\text{anti-cl}} = +1 / 3$. We have thus obtained a non-integer value of the electric charge to be assigned to a quark, which is a component of hadron.

Consider the $((\Delta\hat{\lambda}_3))$ matrix difference of the matrices $[((\hat{\lambda}_3)), ((Q))]$

$$\Delta\hat{\lambda}_3 = \left(\hat{\lambda}_3 \hat{Y} - \hat{Q}\right) = \begin{pmatrix} (1/3) & 0 & 0 \\ 0 & -(1/3) & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -(2/3) & 0 & 0 \\ 0 & +(2/3) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{124}$$

The electric charge calculated with this matrix gives the other value of charge associated with the quark, but it is noted that the matrices $((\hat{\lambda}_3))$ and $((\Delta\hat{\lambda}_3))$ are relatively "opposite" and therefore give opposite values of electric charge $Q \rightarrow Q' = -Q$.

We will then write:

$$Q'_{cl} = \left(\left(\hat{\Psi}^{+}\right)\right)_{cl} \hat{\lambda}_3 \left(\left(\hat{\Psi}\right)\right)_{cl} = \begin{pmatrix} \left(\hat{\bullet}^{+}_{el} e^{i\rho} + \hat{o}^{+}_{in} e^{i\rho^\circ}\right) \\ \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right) \\ \left(\hat{\bullet}^{+}_{el} e^{i\sigma} + \hat{o}^{+}_{in} e^{i\sigma^\circ}\right) \end{pmatrix} \begin{pmatrix} -2/3 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) \\ \left(\hat{\bullet}^{+}_{el} e^{-i\alpha} + \hat{o}^{+}_{in} e^{-i\alpha^\circ}\right) \\ \left(\hat{o}_{el} e^{-i\sigma} + \hat{\bullet}_{in} e^{-i\sigma^\circ}\right) \end{pmatrix}$$

(125)

$$Q'_{cl} = (2/3) \begin{pmatrix} \left(\hat{\bullet}^{+}_{el} e^{i\rho} + \hat{o}^{+}_{in} e^{i\rho^\circ}\right) \\ \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right) \\ \left(\hat{o}^{+}_{el} e^{i\sigma} + \hat{\bullet}^{+}_{in} e^{i\sigma^\circ}\right) \end{pmatrix} \begin{pmatrix} -\left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) \\ \left(\hat{\bullet}^{+}_{el} e^{-i\alpha} + \hat{o}^{+}_{in} e^{-i\alpha^\circ}\right) \\ 0 \end{pmatrix} =$$

$$= (2/3)\left\{-\left(\hat{\bullet}^{+}_{el} e^{i\rho} + \hat{o}^{+}_{in} e^{i\rho^\circ}\right)\left(\hat{o}_{el} e^{-i\rho} + \hat{\bullet}_{in} e^{-i\rho^\circ}\right) + \left(\hat{o}_{el} e^{i\alpha} + \hat{\bullet}_{in} e^{i\alpha^\circ}\right)\left(\hat{\bullet}^{+}_{el} e^{-i\alpha} + \hat{o}^{+}_{in} e^{-i\alpha^\circ}\right)\right\} =$$

$$= (2/3)\left\{-\left(\hat{\bullet}^{+}_{el} \hat{\bullet}_{in} e^{i(\rho-\rho^\circ)} - \hat{\bullet}_{in} \hat{\bullet}^{+}_{el} e^{i(\alpha^\circ-\alpha)}\right) - \left(\hat{o}^{+}_{in} \hat{o}_{el} e^{i(\rho^\circ-\rho)} - \hat{o}_{el} \hat{o}^{+}_{in} e^{i(\alpha-\alpha^\circ)}\right)\right\}$$

$$= (2/3)\left\{-\left[\hat{\bullet}^{+}_{el}, \hat{\bullet}_{in}\right] e^{i(\pi/2)} - \left[\hat{o}^{+}_{in}, \hat{o}_{el}\right] e^{-i(\pi/2)}\right\} = (2/3)\left\{-i\left(\frac{i}{2}\right) + i\left(\frac{-i}{2}\right)\right\} = (2/3)$$

$$Q_{cl} = -Q'_{cl} = -(2/3)$$

(126)

We have thus obtained IQO electrically charged but with non-integer values: (± 1/3, ± 2/3).

6 Configuration of the structure of a quark and a hadron (proton)

The Hypothesis affirms that the structure of a quark (Q-IQO) consists of three Sub-IQO's (see Eq. 9.5), elastically coupled by "lines", in which a current of semi-quanta (current isospin) "flows".
This latter aspect implies that the line, in which the s-q flows, consists (see the representative model of a IQO) in a linear sequence of elastically coupled IQOìs (a "chain"). A chain of coupled IQO's may well be a "line" field on which is moving the representative quantum of the particle-field. (See the following figure 15)

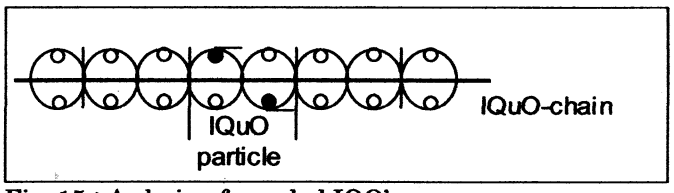


Fig. 15 : A chain of coupled IQO's

Three oscillating (sub-IQO), coupled mutually by three chains, make up a closed structure. There is only one "geometric" possibility: that of a triangle.

Therefore we deem that the quark is given by one "triangular" structure of three sub-IQO's (IQO-vertices) coupled to each other through IQO chains ("sides"), in which a current of semi-quanta flows.
The variable descriptive of the s-q current is the Isospin (T), whose projection (T_3) is connected to the rotation direction of the current. We have the following correspondence:

- Clockwise ⇔ $T_3 = -1/2$
- Anticlockwise ⇔ $T_3 = +1/2$

The variable descriptive of the phase rotation in IQO-vertices is the electric charge Q

- Clockwise ⇔ (-Q)
- Anticlockwise ⇔ (+Q)

From the triangular structure of the couplings, it follows that the direction of rotation of the s-q current along the "sides" shall be concordant with the direction of rotation of the phase of the IQO-vertices. This expresses the correlation (Q ⇔T3). Also to its structure it shall be associated a descriptive variable that we think to be the Y (hypercharge). It is evident that the probability of finding the quantum of a particle inside of a structure depends on the latter. As the value of the electric charge Q has been associated with the probability of detecting the representative quantum of the quark, it follows the (Q ⇔ Y) correlation.
The correlation (Q ⇔ Y⇔T_3) is now:

- Clockwise ⇔ Q = [(-1/3, -2/3)]
- Anticlockwise ⇔ Q [(+1/3, +2/3)]

We can ask ourselves in what way the Y is related to the structure.
A triangular structure of IQO-V coupled suggests that more chains of IQO's get involved, and therefore it should exist the possibility to realize a "transverse" coupling between separate chains. Chains involved in the triangular set defining a quark can be in number of two or three at the most (see Fig. 16 and 17)

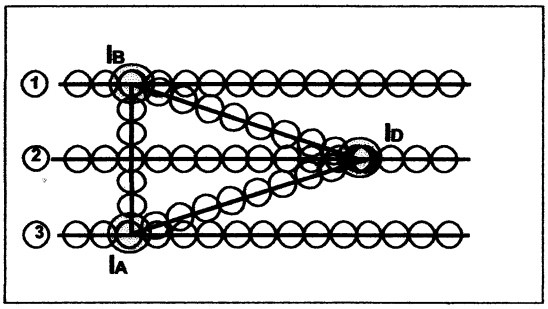


Fig. 16

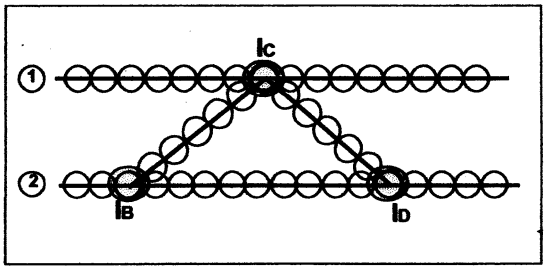


Fig.17

In the first case the probability of finding the quantum is 1/3 because the chains are three in number, while in the second case the probability is 2/3 because the chains are two. **We can conjecture that the first figure shows a quark (d), while the second shows the quark (u).** From everything we affirmed above, we can say that the hypercharge Y is the physical variable indicating the triangular structure of coupling of Sub-IQO vertices to which we associate the value 1/3: so as to say that there is one particle out of three vertices. Also, noting that a triangle in the mirror (P) undergoes inversion right-left, we assign the value of the hypercharge Y = -1/3 to antimatter. We note that the direction of rotation of any rotating object reflected in the mirror is reversed, thus for both the isospin and the electrical charge they are changing the sign: [($\pm T_3$ ➔ $\mp$ T_3);]. It is then obvious that you must also have a reversal of the value of hypercharge ($\pm Y$ ➔$\mp Y$). We have thus suggested physical meanings about what we describe with electric charge Q, with isospin T and hypercharge Y. If we consider a proton as a particle composed of quarks (u, u, d), then you can guess that the proton's structure must be a "geometric" combination of three triangles, as is effectively a "pentagon":

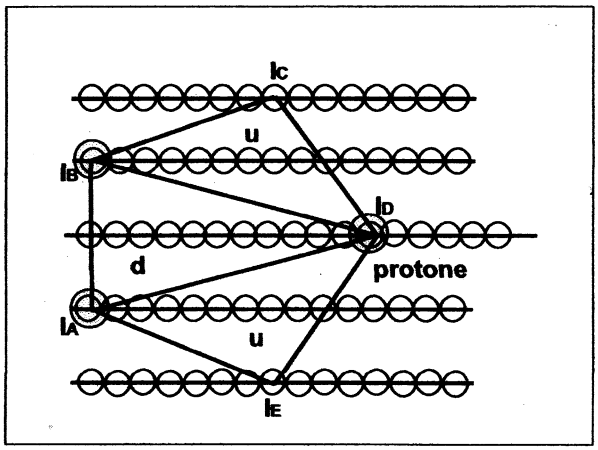


Fig. 18: The proton's structure

While considering a "static model" of the coupling of the quarks that build a proton, it can be naturally assumed that the proton could very well propagate along the five chains IQO keeping unchanged its shape.
In support of the Hypothesis of the Structure, we see that the IQO-Vertices (I_A,I_B,I_D) are the IQO's with which a photon is coupled and an more how experimentally, in diffusion processes that investigate the internal structure of the proton, we highlight **three centers diffusers** with triangular structure.
Still considering diffusion processes, one can conjecture that the diagonal AD represents precisely the Compton wavelength assigned to the proton. At the time when you give the proton a pentagonal structure, it is evident that the diagonals "supporting" the structure [(I_AI_D),(I_BI_D)] are in a "golden" ratio ($\phi \approx 1,618$) → [(AD/AB) = ϕ]. This also implies that **the quarks (u, d) are "golden" triangles.**
A geometric property of the golden triangles of type (u) or (d) is that each "aureus" triangle consists in its turn of two golden triangles of the two types inscribed in it. Because the supporting diagonals in the proton are essential for its structure, it's conjectured that the side AD in the quark (u) can be coincident with its Compton wavelength ($\lambda_p = \hbar/mc$). For analogy, we can believe that the same diagonal AD can represent the transverse coupling "supporting", in the structure of the component sub-IQO, the quark (u) except for a scale ratio. Always in geometric terms, it is deduced that the side AB in (d) can represent the transverse coupling "supporting", in the structure of the component sub-IQO, the quark (d) except for a scale ratio.

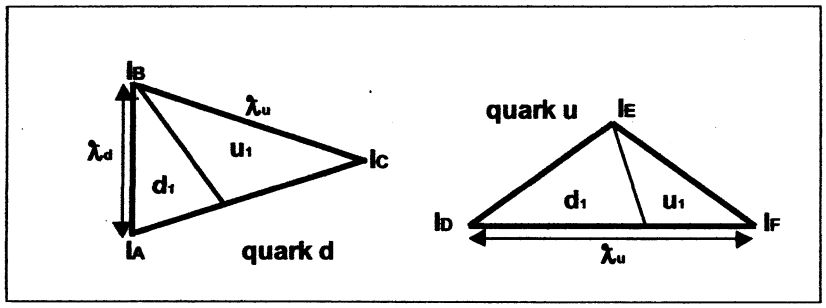


Fig.19

Insofar, the ratio between the masses of the two fundamental quarks will be: **[(m_d/m_u) = $\phi \approx 1,618$] with ($m_d > m_u$). It is evident that this original static model of the quarks can have validity if it is found that the Compton wavelength of the proton, to less than a scale factor, coincides with that of the quark (u) → (λ_p) ≡ (λ_u).**
For further confirmation of this idea, we note that the ratio between the **wavelength Compton of the Planck's particle and the one of proton is (to less than a scale factor) : [(λ_p/λ_{pl}) ≈1/2 ϕ^2].**

Final considerations

As it has been underlined in this paper, the semi-quanta structure of an oscillator and the coupling mechanism of two IQO's determine a set of situations that is strongly analogous to the typical phenomenology of electrically charged particles and of the process of pair creation.
We think that this original approach represents something staying at the base of elementary particle creation and of their behavior in relation to the electric charge. So we might be in the presence of a model of structure of the quantum fields, able of explaining in a clearer way the complex nature of particles and their interactions. To support this model, we underline the following aspects:

1) connecting the sign of the electric charge to the direction of phase rotation [4] highlights the correlation between the fields with electric charge and the invariance for gauge transformations of their interactions.
2) if we associate to the electric charge and to isospin T the internal rotations, then we can give to quarks an internal structure of quantum coupled oscillators (IQO)
3) giving an internal structure to the quark, allows us to interpret in terms of probability the non-integer electric charges.

Some aspects of this study can prelude to a new key for approaching some problems still open in modern physics :

- the phase "interaction" between two IQO's could help to understand better the physical process of reduction of the wave function.
- Since massless particles, with electric charge, don't exist, the coupling process of two Φ-IQO's, that leads to the creation of two Ψ-IQO's with electric charge, could contribute to a better explanation explain of the creation of the massive particles.
- The mechanism in which a gauge field **(Φ-IQO)** regularizes the form of an "irregular" field **Ψ-IQO**, reminds us of electric charges (like an electron) surrounded by clouds of virtual photons.
- The structure by semi-quanta of the field oscillators Ψ-IQO and Φ-IQO could give us some information about the structure of particles and about fundamental difference between Fermions and Bosons (as well as as the one between Leptons and Quarks).
- The sub-IQO model of quarks can help defining the structure of hadrons and better explain the dynamics of the coupling of the quarks that determine strong interactions
- The s-q currents may be related to gluon currents.
- The quark confinement would have a "structural" (and not dynamic) explanation.

In conclusion, the structure by semi-quanta of the field oscillator could be a key to understand the nature of particles, their formation and their interactions.

References:

[1] : A.S. Davidov in "*Meccanica Quantistica*", Edizioni Mir 1981 Italy, pp. 243-247

[2] : J. J. Sakurai in "*Modern Quantum Mechanics*", Benjamin /Cummings Publishimng Company , Inc
1985 , USA

[3] : H. Feshbach, F. Villars in "*Elementary Relativistic Wave Mechanics of Spin 0 and Spin ½*". Rev. Mod.
Phys. 30, 24 (1958)

[4] : G. Guido " *The substructure of a quantum oscillator field*", in **arxiv**.org/pdf/1208.0948

[5] : "*Wikipedia*", in internet

INDEX
HADRONIC JOUNAL, VOLUME 36, 2013

SOME THOUGHTS ON SPECIAL RELATIVITY, 1
Jeremy Dunning-Davies
Institute for Theoretical Physics and Advanced Mathematics (IFM)
Einstein-Galilei, Via Santa Gonda, 14 - 59100 Prato, Italy
and
Institute for Basic Research, Palm Harbor, FL, U.S.A.

RETHINKING THE FORMAL METHODOLOGY (I): WAVE-VORTEX ESSENCE OF THE SUBSTANCE, 9
George Kirakosyan
Physics Department, State Engineering University
Yerevan, Armenia, P.O. Box 42, Dubai, UAE

SANTILLI'S NEW MATHEMATICS FOR CHEMISTS AND BIOLOGISTS. AN INTRODUCTORY ACCOUNT, 51
Anil A. Bhalekar
R.T.M. Nagpur University, Department of Chemistry
Amravati Road Campus, Nagpur – 440 033, India

GENO-BRAGG'S LAW AND GENO-EINSTEIN'S GENERAL RELATIVITY THEORY FOR QUASICRYSTAL STRUCTURES, 149
Alexander O.E. Animalu
Department of Physics and Astronomy
University of Nigeria, Nsukka
and
Enugu State, Nigeria & International Centre for Basic Research
20 Limpopo Street, FHA, Maitama, Abuja, Nigeria

DYNAMIC ISO-TOPIC LIFTING WITH APPLICATION TO FIBONACCI'S SEQUENCE AND MANDELBROT'S SET, 167
Nathan O. Schmidt
Department of Mathematics, Boise State University
1910 University Drive
Boise, ID 83725 USA

ACTIVE SITES, COMPLEXITY STRUCTURES AND HYPERSTRUCTURES IN CATALYSTS, 177
M. Piumetti
Department of Applied Science and Technology
Politecnico di Torino, 10129 Torino, Italy
N. Lygeros
LGPC (UMR 5285), Université de Lyon
69616, Villeurbanne, France

DARK ENERGY, INERTIA AND MACH'S PRINCIPLE, 197
C. Sivaram
Indian Institute of Astrophysics, Bangalore – 560 034, India
Kenath Arun
Christ Junior College, Bangalore – 560 029, India
MAGNETIC EFFECT OF NON-COMMUTATIVITY, 205
B.G. Sidharth
G.P. Birla Observatory & Astronomical Research Centre
B.M. Birla Science Centre
Adarsh Nagar, Hyderbad – 500 063 India
C.V. Aditya
University of Stuttgart, Germany
MADELBROT ISO-SETS: ISO-UNIT IMPACT ASSESSMENT, 211
Reza Katebi
Department of Physics, California State University
Fullerton, 800 North State College Boulevard, Fullerton, CA 92831 USA
Nathan Schmidt
Department of Mathematics, Boise State University
1910 University Drive, Boise, ID 83725 USA
ISO-GALOIS FIELDS, 225
Arun S. Muktibodh
Department of Mathematics, Mohota College of Science
Nagpur-440009 India
ON THE MASS-ENERGY RELATIONSHIP $E=mc^2$, 237
John Shim
FERMIODYNAMICS THROUGH GENERALIZED ELECTRODYNAMICS, 243
Sushama V. Vaidya
Physics Department S.B. College of Science
Aurangabad – 431001, India
M.T. Teli
Physics Department, Dr. Babasaheb Ambedkar
Aurangabad – 431004, India (Retd.)
ON GENERALIZED JORDAN PREDERIVATIONS AND GENERALIZED PREDERIVATIONS OF JORDAN LIE SUPERALGEBRAS, 265
Jianrong Peng, Yao Ma, Liangyun Chen
Department of Mathematics
Northeast Normal University
Changchun 130024, China

HADRONIC CHEMISTRY AND BINDING ENERGIES, 283
Sudhakar S. Dhondge
P.G. Department of Chemistry, S.K. Porwal College
Kamptee, Nagpur - 441 001, India
Anil A. Bhalekar
Department of Chemistry
R.T.M. Nagpur Univ., Amravati Rd. Campus, Nagpur – 440 033, India
And Thunder Fusion Corporation, U.S.A.
And Institute for Theoretical Physics and Advanced Mathematics, Italy
DOES AN INITIAL NON PATHEOLOGICAL SINGULARITY AFFECT DIFFERENTIATION BETWEEN EITHER A SINGLE UNIVERSE OR A MULTIVERSE?, 327
Andrew Beckwith
Chongqing University Department of Physics, Chongqing, PRC, 400044
ON THE INTERPRETATION OF THE LORENTZ TIME TRANSFORMATION, 345
John Shim
SANTILLI'S LIE-ADMISSIBLE MECHANICS. THE ONLY OPTIO COMMENSURATE WITH IRREVERSIBILITY AND NONEQUILIBRIUM THERMODYNAMICS, 349
Anil A. Bhalekar
Department of Chemistry, R.T.M. Nagpur Univ., Amravati Rd. Campus,
Nagpur – 440 033, India
And Thunder Fusion Corporation, U.S.A.
And Institute for Theoretical Physics and Advanced Mathematics, Italy
SANTILLI'S ISOTOPIC, GENOTOPIC, HYPER-STRUCTURAL AND ISODUAL FOUR DIRECTIONS OF TIME AND NONEQUILIBRIUM THERMODYNAMICS, 387
Anil A. Bhalekar
Department of Chemistry, R.T.M. Nagpur Univ., Amravati Rd. Campus,
Nagpur – 440 033, India
And Thunder Fusion Corporation, U.S.A.
And Institute for Theoretical Physics and Advanced Mathematics, Italy
ORIGIN OF QUARKS THROUGH FERMIODYNAMICS, 407
Sushama V. Vaidya
Physics Department, S.B. College of Science, Aurangabad – 431001, India
M.T. Teli
Physics Department, Dr. Babasaheb Ambedkar, Marathwada University
Aurangabad – 431004, India (Retd.)
GRAVITO-ELECTROMAGNETISM EXPLAINED BY THE THEORY OF INFORMATIONS, 415
Antoine Acke
Retired Professor, Department of Industrial Sciences
University College Kaho Sint-Lieven, Gent, Belgium

ON THE INTERPRETATION OF THE LORENTZ TIME TRANSFORMATION, 485
John Shim
DYNAMIC ISO-TOPIC LIFTING WITH APPLICATION TO FIBONACCI'S SEQUENCE AND MANDELBROT'S SET, 489
Nathan O. Schmidt
Department of Mathematics, Boise State University
1910 University Drive, Boise, ID 83725, USA
THE MISSING MASS OF THE MILKY WAY GALAXY, 499
Kenneth Dalton
SANTILLI'S NEW FUELS AS SOURCES OF CLEAN COMBUSTION, 507
Indrani B. Das Sarma
Department of Applied Chemistry, Jhulelal Institute of Technology
Lonara, Nagpur – 441 111, India
AN INTRODUCTORY ACCOUNT OF SANTILLI'S IRS, IBS AND NIS, 529
Ritesh L. Kohale
Department of Physics, R.T.M. Nagpur University
Nagpur – 440033, India
MADELBROT ISO-SETS: ISO-UNIT IMPACT ASSESSMENT, 541
Reza Katebi
Department of Physics, California State University
800 North State College Boulevard, Fullerton, CA 92831, USA
Nathan O. Schmidt
Department of Mathematics, Boise State University
1910 University Drive, Boise, ID 83725, USA
THE GRAVITATIONAL ENERGY OF THE UNIVERSE, 555
Kenneth Dalton
General Delivery, Bang Saphan Yai 77140
Prachuap Kiri Khan, Thailand
THE MAGNEHYDROGEN™ IN HYDRONIC CHEMISTRY, 565
Sangesh P. Zodape
Department of Chemistry, Visvesvaraya National Institute of Technology
Nagpur – 440 010, India
Anil A. Bhalekar[2]
Department of Chemistry
R.T.M. Nagpur Univ., Amravati Rd. Campus, Nagpur – 440 033, India
And Thunder Fusion Corporation, U.S.A.
And Institute for Theoretical Physics and Advanced Mathematics, Italy
INTRODUCTION TO THE NEW SPECIES OF THE MAGNECULES OF HADRONIC CHEMISTRY, 593
Chandrashekhar P. Pandhurnekar
Department of Applied Chemistry, Ramdeobaba College of Engineering and Management, Nagpur – 440 013, India

COMMENT ON "WHAT THE INFORMATION LOSS IS NOT", 625
Qing-yu Cai, Baocheng Zhang, Miangsheng Zhan
Wuhan Institute of Physics and Mathematics
Chinese Academy of Sciences, Wuhan 430071
People's Republic of China
Li You
Department of Physics, Tsinghua University,People's Republic of China
A CYCLIC MODEL OF GALAXY EVOLUTION WITH BARS, 633
Peter A. Jackson
RAS, AAS, Orchard House Hillside Rd., Whitstable CT5 3EX, UK
John S. Minkowski
Baltimore, MD 21204, USA
ELECTROMAGNETISM EXPLAINED BY THE THEORY OF INFORMATONS, 677
Antoine Acke
Retired Professor, Department Industrial Sciences
University College Kaho Sint-Lieven, Gent – Belgium